Introduction to Geophysics

[ES 3004 / ES 7020]

Semester 2, AY2025–2026

Nanyang Technological University

Tutorial
Yield Strength Envelope (YSE) of the Continental Lithosphere

1. Physical setting

Rocks in the lithosphere can deform by different mechanisms depending on depth, pressure, temperature, and timescale:

- **Brittle (frictional) regime:** strength increases with confining pressure.
- **Ductile (viscous) regime:** strength depends strongly on temperature and strain rate.

A *Yield Strength Envelope* (YSE) combines these mechanisms to estimate the **maximum differential stress** the lithosphere can sustain at each depth. In this tutorial you will build a simple YSE for a **continental lithosphere** with three layers:

- Upper crust: $0 \le z \le z_{uc}$,
- Lower crust: $z_{uc} < z \le z_{lc}$,
- Lithospheric mantle: $z > z_{lc}$.

You will run two end-member models for the **lower crust**:

- **Weak lower crust** (easier viscous flow),
- **Strong lower crust** (harder viscous flow).

All other parameters remain the same.

2. What you are building

Your MATLAB script will compute and plot three curves versus depth:

1. brittle strength $\sigma_{\text{brittle}}(z)$,
2. ductile strength $\sigma_{\text{ductile}}(z)$,
3. yield envelope $\sigma_{\text{yield}}(z) = \min(\sigma_{\text{brittle}}, \sigma_{\text{ductile}})$.

Important: units. Use SI units in the calculations:

$$z \text{ [m]}, \quad T \text{ [K]}, \quad \sigma \text{ [Pa]}, \quad \dot{\varepsilon} \text{ [s}^{-1}\text{]}.$$

Convert stress to MPa only for plotting:

$$1 \text{ MPa} = 10^6 \text{ Pa}.$$

3. Brittle strength (frictional law)

We use the simplified frictional form used in lecture:

$$\sigma_{\text{brittle}}(z) = c + \mu\, \rho(z)\, g\, z, \tag{1}$$

where c is cohesion (Pa), μ is friction coefficient, $\rho(z)$ is the density of the layer at depth z, and g is gravity.

In a layered lithosphere, use *piecewise densities*:

$$\rho(z) = \begin{cases} \rho_{uc}, & 0 \le z \le z_{uc}, \\ \rho_{lc}, & z_{uc} < z \le z_{lc}, \\ \rho_m, & z > z_{lc}. \end{cases}$$

4. Ductile strength (power-law creep)

We model ductile flow with a power-law creep relation:

$$\dot{\varepsilon} = A\,\sigma^n \exp\left(-\frac{Q}{RT}\right), \tag{2}$$

where $\dot{\varepsilon}$ is strain rate (s^{-1}), σ is differential stress (Pa), A is a prefactor (Pa^{-n} s^{-1}), n is the stress exponent, Q is activation energy (J/mol), R is the gas constant, and T is temperature (K).

Solve Eq. (2) for stress:

$$\sigma_{\text{ductile}}(z) = \left[\frac{\dot{\varepsilon}}{A}\,\exp\left(\frac{Q}{R\,T(z)}\right)\right]^{1/n}. \tag{3}$$

5. Yield Strength Envelope

At each depth, the lithosphere yields by the *weaker* mechanism (lower required stress). Thus,

$$\sigma_{\text{yield}}(z) = \min(\sigma_{\text{brittle}}(z),\ \sigma_{\text{ductile}}(z))\,. \tag{4}$$

6. Temperature profile (geotherm)

To evaluate $\sigma_{\text{ductile}}(z)$ you must specify $T(z)$. Use the following **piecewise geotherm**:

- Surface temperature: T_{surf}.
- At the Moho ($z = z_{lc}$): T_{moho}.
- At the lithosphere–asthenosphere boundary (LAB), depth $z = z_{\text{LAB}}$: T_{LAB}.

Define:

$$T(z) = \begin{cases} T_{\text{surf}} + (T_{\text{moho}} - T_{\text{surf}})\,\dfrac{z}{z_{lc}}, & 0 \le z \le z_{lc}, \\ T_{\text{moho}} + (T_{\text{LAB}} - T_{\text{moho}})\,\dfrac{z - z_{lc}}{z_{\text{LAB}} - z_{lc}}, & z_{lc} < z \le z_{\text{LAB}}, \\ T_{\text{LAB}}, & z > z_{\text{LAB}}. \end{cases}$$

7. Parameter values to use (copy exactly into your script)

Global constants

$$g = 9.81\ \text{m/s}^2, \qquad R = 8.314\ \text{J/(mol K)}, \qquad \dot{\varepsilon} = 10^{-15}\ \text{s}^{-1}.$$

$$\mu = 0.6, \qquad c = 0\ \text{Pa}.$$

Layer boundaries

$$z_{uc} = 20\ \text{km}, \qquad z_{lc} = 35\ \text{km}, \qquad z_{\max} = 150\ \text{km}, \qquad z_{\text{LAB}} = 120\ \text{km}.$$

Densities

$$\rho_{uc} = 2700\ \text{kg/m}^3, \qquad \rho_{lc} = 2900\ \text{kg/m}^3, \qquad \rho_m = 3300\ \text{kg/m}^3.$$

Geotherm values

$$T_{\text{surf}} = 273\ \text{K}, \qquad T_{\text{moho}} = 873\ \text{K}\ (600^\circ\text{C}), \qquad T_{\text{LAB}} = 1573\ \text{K}\ (1300^\circ\text{C}).$$

Creep parameters (upper crust and mantle; fixed)

$$\text{Upper crust:} \quad A_{uc} = 1.0 \times 10^{-28}\ \text{Pa}^{-\text{n}}\,\text{s}^{-1}, \quad n_{uc} = 4.0, \quad Q_{uc} = 223 \times 10^3\ \text{J/mol}.$$

$$\text{Mantle lithosphere:} \quad A_m = 1.0 \times 10^{-28}\ \text{Pa}^{-\text{n}}\,\text{s}^{-1}, \quad n_m = 3.5, \quad Q_m = 520 \times 10^3\ \text{J/mol}.$$

Lower crust: run TWO cases (weak vs strong)

- **Case 1: Weak lower crust** (easier flow; lower ductile strength)

$$A_{lc} = 6.7 \times 10^{-20}\ \text{Pa}^{-\text{n}}\,\text{s}^{-1}, \quad n_{lc} = 2.3, \quad Q_{lc} = 135 \times 10^3\ \text{J/mol}.$$

- **Case 2: Strong lower crust** (harder flow; higher ductile strength)

$$A_{lc} = 1.0 \times 10^{-28}\ \text{Pa}^{-\text{n}}\,\text{s}^{-1}, \quad n_{lc} = 3.5, \quad Q_{lc} = 240 \times 10^3\ \text{J/mol}.$$

8. Questions for students

1. Why does the brittle strength increase with depth in Eq. (1)?
2. In Eq. (3), explain qualitatively how increasing temperature affects ductile strength.
3. Compare the yield envelopes for weak vs strong lower crust:
   - Where is the lithosphere weakest in each case?
   - Does the "weak zone" (if any) sit in the lower crust or elsewhere?
4. How would the ductile curve change if $\dot{\varepsilon}$ increased by a factor of 10?
5. Identify approximately the depth where $\sigma_{\text{brittle}} = \sigma_{\text{ductile}}$ (brittle–ductile transition) in the crust for both weak and strong lower crust cases.