Introduction to Geophysics

[ES 3004 / ES 7020]

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Nanyang Technological University

Tutorial
2D Thermal Convection

1. Physical motivation

The Earth continuously loses heat to space. This heat originates from primordial formation energy and radiogenic decay. The way heat is transported inside the mantle controls plate tectonics, mantle plumes, and the long-term thermal evolution of the planet.

There are two fundamental modes of heat transport:

- **Conduction**: heat diffuses through a material without bulk motion.

- **Convection**: heat is transported by moving material driven by buoyancy.

In this tutorial, we numerically explore the transition between conduction and convection in a simplified 2D model.

2. Governing equations

We use the **Boussinesq approximation**, which assumes that density variations are small and only important in the buoyancy term. In practice, this means that density is treated as constant everywhere. This approximation is valid when temperature variations produce only small relative density changes (as in the mantle).

Heat equation with advection

$$\frac{\partial T}{\partial t} = \kappa \nabla^2 T - \mathbf{u} \cdot \nabla T, \tag{1}$$

where T is temperature, κ thermal diffusivity, and $\mathbf{u}$ the velocity field.

3. The Rayleigh number

The Rayleigh number controls whether convection occurs:

$$Ra = \frac{\rho g \alpha \Delta T H^3}{\kappa \eta}. \tag{2}$$

Parameters:

- ρ density

- g gravity

- α thermal expansion

- ΔT temperature difference

- H layer thickness

- κ thermal diffusivity

- η viscosity

Key concept:

$$Ra \ll Ra_{\mathrm{crit}} \Rightarrow \text{conduction}$$

$$Ra \gg Ra_{\mathrm{crit}} \Rightarrow \text{convection}$$

The Earth's mantle has $Ra \sim 10^7$–10^8, so convection must occur.

4. Numerical setup

The MATLAB script:

- Discretizes a 2D box using finite differences.

- Applies fixed temperatures at top and bottom.

- Uses insulated side boundaries.

- Runs two cases:
    - Low Ra (pure conduction),
    - High Ra (convection).

5. Conduction regime (low Rayleigh number)

For small Ra:

- No organized flow develops.

- Temperature evolves toward a linear vertical gradient.

- Heat transport is purely diffusive.

Physically, viscosity dominates buoyancy forces.

6. Convection regime (high Rayleigh number)

For large Ra:

- Convective rolls appear.

- Hot material rises.

- Cold material sinks.

- Isotherms become distorted.

Now buoyancy overcomes viscous resistance, and the system self-organizes into flow patterns.

7. Critical aspects you must understand

- Convection is not imposed, it emerges from buoyancy.

- Increasing H strongly increases Ra ($Ra \propto H^3$).

- Viscosity suppresses convection.

- The mantle behaves as a very slow, highly viscous convecting fluid.

- Plate tectonics is a surface expression of mantle convection.

8. Conceptual questions

1. Why does doubling the layer thickness increase the likelihood of convection?

2. What happens to Ra if viscosity increases by a factor of 10?

3. Why is the mantle convecting even though it is solid?

4. Which parameter in Ra is most uncertain for the real Earth?

9. Key takeaways

- Heat loss drives mantle dynamics.

- The Rayleigh number determines the transition from conduction to convection.

- Convection naturally generates upwellings and downwellings.

- A simple 2D model already captures first-order mantle behavior.

This tutorial demonstrates how basic physics explains large-scale planetary dynamics.