Introduction to Geophysics

[ES 3004 / ES 7020]

Semester 2, AY2025–2026

Nanyang Technological University

Tutorial
Mohr Circle Construction from a Stress Tensor

1. Physical setting

The state of stress at a point inside the Earth is described by the stress tensor,

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix}. \tag{1}$$

Mohr's circle provides a graphical representation of the normal and shear stress acting on any plane through that point. It is widely used in structural geology, rock mechanics, and seismology because it allows us to:

- identify the principal stresses,
- compute the maximum shear stress,
- visualize how stresses act on planes of arbitrary orientation.

Here we restrict ourselves to a 2D stress state in the x–y plane:

$$\boldsymbol{\sigma}_{2D} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{xy} & \sigma_{yy} \end{pmatrix}.$$

2. Stress transformation on an arbitrary plane

Consider a plane whose outward normal makes an angle θ with the x-axis. The transformed stresses acting on that plane are:

$$\sigma_n = \frac{\sigma_{xx} + \sigma_{yy}}{2} + \frac{\sigma_{xx} - \sigma_{yy}}{2}\cos(2\theta) + \sigma_{xy}\sin(2\theta), \tag{2}$$

$$\tau = -\frac{\sigma_{xx} - \sigma_{yy}}{2}\sin(2\theta) + \sigma_{xy}\cos(2\theta), \tag{3}$$

where σ_n is the normal stress and τ the shear stress on the plane.

3. Constructing Mohr's circle

Mohr's circle is obtained by eliminating θ between the above equations. The result is the Cartesian equation of a circle:

$$(\sigma_n - \sigma_c)^2 + \tau^2 = R^2, \tag{4}$$

with center

$$\sigma_c = \frac{\sigma_{xx} + \sigma_{yy}}{2}, \tag{5}$$

and radius

$$R = \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2}. \tag{6}$$

The principal stresses correspond to the points where $\tau = 0$:

$$\sigma_{1,2} = \sigma_c \pm R. \tag{7}$$

The maximum shear stress is simply equal to the radius:

$$\tau_{\max} = R. \tag{8}$$

4. MATLAB exercise

In this tutorial you will:

1. Input a 2D stress tensor $(\sigma_{xx}, \sigma_{yy}, \sigma_{xy})$.
2. Compute the center σ_c and radius R of Mohr's circle.
3. Compute the principal stresses σ_1 and σ_2.
4. Plot Mohr's circle and annotate principal stress points.

5. Suggested stress tensor

Use this example for your MATLAB script:

$$\sigma_{xx} = 150 \text{ MPa}, \qquad \sigma_{yy} = 250 \text{ MPa}, \qquad \sigma_{xy} = 40 \text{ MPa}.$$

6. Questions for students

1. Which plane orientation corresponds to the maximum shear stress?
2. How do the principal stresses compare to the original tensor components?
3. How does increasing σ_{xy} change the radius of Mohr's circle?