Introduction to Geophysics

Assignment: Onset of Convection and the Rayleigh Number

In this assignment you will investigate the physical conditions required to trigger convection in a viscous fluid layer. You are **not allowed to use water**. Instead, choose one of the following fluids:

- Olive oil

- Maple syrup

- Honey

You must search for realistic thermophysical properties from reliable sources.

Problem 1: Critical temperature difference for convection

Convection in a horizontal fluid layer heated from below begins when the Rayleigh number exceeds the critical value:

$$Ra = \frac{\rho g \alpha \Delta T H^3}{\kappa \eta}$$

where:

- ρ = density

- g = gravity

- α = thermal expansion coefficient

- ΔT = temperature difference between bottom and top

- H = fluid thickness

- κ = thermal diffusivity

- η = viscosity

Convection starts when:

$$Ra \geq Ra_c \approx 1700.$$

Consider a glass of height:

$$H = 0.15\,\mathrm{m}$$

with top temperature:

$$T_{\mathrm{top}} = 20\,^\circ\mathrm{C}.$$

Tasks:

(a) Find realistic values for the following properties of your chosen fluid:

$$\rho, \quad \alpha, \quad \kappa, \quad \eta$$

at approximately room temperature.
Provide the source of your values.

(b) Rearrange the Rayleigh number formula to solve for the critical temperature difference ΔT required to reach Ra_c.

(c) Compute the required bottom temperature:

$$T_{\text{bottom}} = T_{\text{top}} + \Delta T.$$

Problem 2: Effect of layer thickness

Now repeat the calculation for a taller glass:

$$H = 0.30 \, \text{m}$$

(a) Compute the new ΔT.

(b) Compare the two cases and explain why the difference is large.

Problem 3: Why mantle convection exists

The Earth's mantle has a thickness of roughly:

$$H_{\text{mantle}} \sim 2800 \, \text{km}.$$

The mantle viscosity is extremely large ($\eta \sim 10^{21}$–10^{23} Pa s). Explain why convection is nevertheless possible.