Introduction to Geophysics

Assignment: Rheology of Rocks (Brittle, Ductile, Viscoelastic)

Problem 1: Brittle strength with depth (Mohr–Coulomb, simplified)

In lecture we used a simple frictional (brittle) strength law:

$$\sigma_{\text{brittle}}(z) = c + \mu\,(\rho\,g\,z),$$

where z is depth (measured downward), ρ is density, μ is a friction coefficient, and c is cohesion.

Assume:

$$\mu = 0.6, \qquad c = 0, \qquad \rho = 2700\,\text{kg/m}^3, \qquad g = 9.81\,\text{m/s}^2.$$

(a) Compute σ_{brittle} at depths $z = 5\,\text{km}$, $15\,\text{km}$, and $30\,\text{km}$. Give your answers in Pa and MPa.

(b) Briefly explain (2–3 sentences): why does σ_{brittle} increase with depth in this model?

Problem 2: Ductile creep sensitivity to temperature (stress ratio)

A power-law creep relation can be written as:

$$\dot{\varepsilon} = A\,\sigma^n \exp\left(-\frac{Q}{RT}\right).$$

Solving for stress gives (as in lecture):

$$\sigma(T) = \left[\frac{\dot{\varepsilon}}{A}\exp\left(\frac{Q}{RT}\right)\right]^{1/n}.$$

To avoid dealing with A and $\dot{\varepsilon}$, consider the ratio at two temperatures (with the same $\dot{\varepsilon}$ and same material):

$$\frac{\sigma(T_2)}{\sigma(T_1)} = \exp\left[\frac{Q}{nR}\left(\frac{1}{T_2} - \frac{1}{T_1}\right)\right].$$

Assume:

$$Q = 240\,\text{kJ/mol}, \qquad n = 3, \qquad T_1 = 800\,^\circ\text{C}, \qquad T_2 = 1000\,^\circ\text{C}.$$

(a) Convert T_1 and T_2 to Kelvin.

(b) Compute the ratio $\sigma(T_2)/\sigma(T_1)$.

(c) Interpret (2–3 sentences): does hotter rock require higher or lower stress to flow at the same strain rate? Why?

Problem 3: Maxwell time (elastic vs viscous vs viscoelastic)

A Maxwell material (spring + dashpot in series) has a characteristic timescale:

$$\tau_M = \frac{\eta}{E},$$

where E is an elastic modulus and η is viscosity.

Assume:

$$E = 70\,\mathrm{GPa},$$

and consider two viscosities:

$$\eta_1 = 1{\times}10^{19}\,\mathrm{Pa\,s} \quad \text{(warm lower crust / weak)} \qquad \eta_2 = 1{\times}10^{22}\,\mathrm{Pa\,s} \quad \text{(cold lithosphere / strong)}.$$

(a) Compute τ_M for each case in seconds, then convert to years.

(b) For each case, decide whether the material response is mostly **elastic** or mostly **viscous** on:

- a seismic timescale ($t \sim 10\,\mathrm{s}$),
- a post-seismic / glacial timescale ($t \sim 100\,\mathrm{yr}$).

State clearly your criterion (compare t to τ_M).
