Introduction to Geophysics

Assignment: Pressure in the Earth and Isostasy

Instructions. Show all intermediate steps and state clearly any assumptions you make. Use SI units throughout unless otherwise stated. You may use a calculator. Unless specified, take:

$$g = 9.8\,\mathrm{m/s^2}, \qquad R = 6371\,\mathrm{km}.$$

Part A: Pressure Inside the Earth

In this part you will estimate the pressure at the center of the Earth using simple hydrostatic arguments and two different density models.

A1. Constant-density Earth (toy model)

Assume that the Earth has:

- radius $R = 6371\,\mathrm{km}$,
- uniform density $\bar{\rho} = 5500\,\mathrm{kg/m^3}$,
- constant gravitational acceleration $g = 9.8\,\mathrm{m/s^2}$.

(a) Starting from the hydrostatic balance

$$\frac{\partial P}{\partial z} = \rho g$$

with depth z measured downwards from the surface, derive an expression for the pressure at the center of the Earth, P_c, assuming constant ρ and g.

(b) Evaluate P_c numerically in Pascals and in Gigapascals (GPa).

A2. Stratified Earth (layered density model)

Now consider a simplified layered Earth with the following structure. Depths are measured from the surface downward.

Layer	Depth interval (km)	Thickness (km)	Density ($\mathrm{kg/m^3}$)
Crust	0–30	30	2800
Upper mantle	30–660	630	3400
Lower mantle	660–2900	2240	4500
Core	2900–6371	3471	11000

Assume, for simplicity, that $g = 9.8\,\mathrm{m/s^2}$ is constant with depth.

(c) Write an expression for the pressure at the center of the Earth, P_c, as the sum of contributions from each layer under this layered model, using

$$\Delta P_i = \rho_i g h_i,$$

where ρ_i is the density and h_i the thickness of layer i.

(d) Compute P_c numerically for this layered model (in Pa and GPa).

(e) Compare your results from (b) and (d). Which model gives a higher central pressure, and why? Comment briefly, considering that the real Earth is denser toward the center and that g does not actually remain constant with depth.

Part B: Isostasy

In this part you will use a simple Airy isostasy model to relate the height of topography to the thickness of crustal roots.

B1. Background: Airy isostasy derivation

In the Airy model, the crust "floats" on a denser mantle. We assume:

- Crustal density ρ_c,
- Mantle density ρ_m with $\rho_m > \rho_c$,
- A reference column with zero topography and crustal thickness C_0,
- A column with topography height T (e.g. a mountain) and an associated root thickness R.

Let D be a compensation depth deep in the mantle where pressure is equal beneath all columns. In the reference column:

$$P_{\text{ref}} = \rho_c g C_0 + \rho_m g (D - C_0).$$

In the column with topography T and root R, the crustal thickness is $C_0 + T + R$, and the mantle thickness is $D - (C_0 + R)$, so:

$$P_{\text{topo}} = \rho_c g (C_0 + T + R) + \rho_m g \big(D - C_0 - R\big).$$

Isostatic equilibrium requires $P_{\text{ref}} = P_{\text{topo}}$. Show that this leads to the relation

$$\boxed{R = \frac{\rho_c}{\rho_m - \rho_c} T.}$$

(This relation was derived in the lecture; you may use it in the exercise below without re-deriving it.)

B2. Isostasy of a continental plateau

Consider a simple continental crust–mantle system with the following properties:

- Reference crustal thickness (no topography): $C_0 = 35\,\text{km}$,
- Crustal density: $\rho_c = 2800\,\text{kg/m}^3$,
- Mantle density: $\rho_m = 3300\,\text{kg/m}^3$.

Now suppose we form a high plateau with surface elevation $T = 4\,\text{km}$ above sea level.

(a) Using the Airy isostasy relation

$$R = \frac{\rho_c}{\rho_m - \rho_c} T,$$

compute the root thickness R beneath the plateau.

(b) Determine the total crustal thickness beneath the plateau, $C_{\text{plateau}} = C_0 + T + R$.

(c) Assuming sea level as the reference horizontal surface, compute the depth of the Moho (crust–mantle boundary) beneath the plateau below sea level. (Hint: subtract the surface elevation T from the total crustal thickness.)

(d) Briefly compare the Moho depth beneath the plateau to the reference case (no topography). Is the crustal root “large” compared to the surface topography? Comment on the ratio R/T.