Gravity

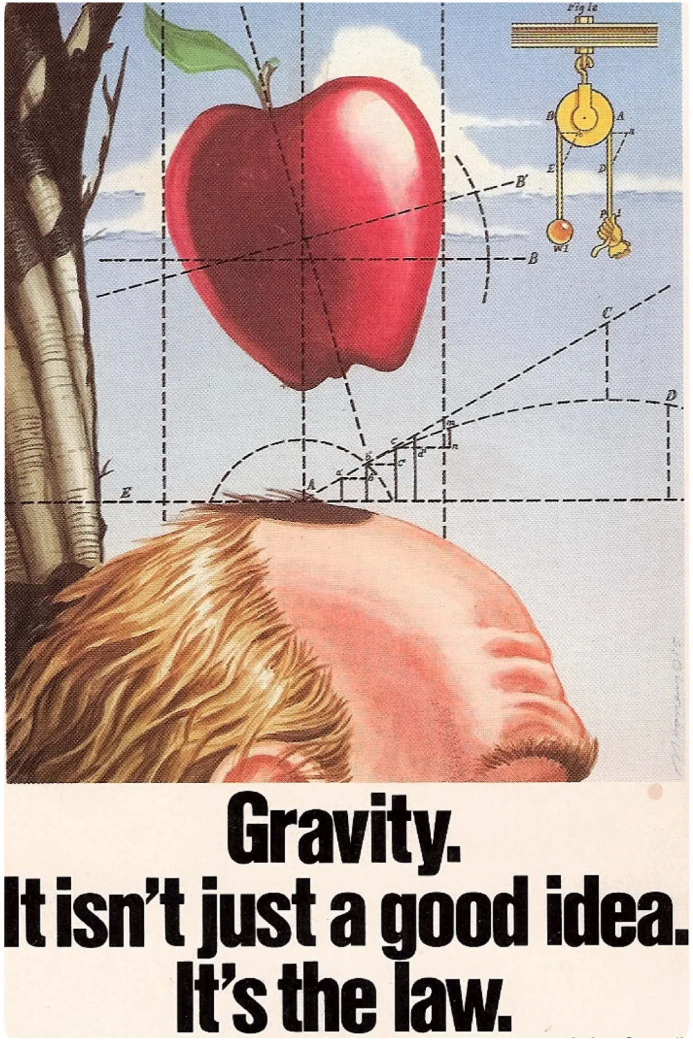
Gravity

Gravity is the fundamental force of attraction between any two objects with mass or energy, pulling them towards each other; it's why things fall to Earth, planets orbit the Sun, and it's described by Newton as a force proportional to mass and distance, and by Einstein as the warping of spacetime by mass and energy. While the weakest of nature's forces, its long reach governs the structure of the universe, from stars to galaxies.

Gravity

“Gravity: It's Not Just a Good Idea, It's the Law” is a popular slogan used on science-themed merchandise and in media, originating from a 1977 idea by Gerry Mooney, highlighting gravity as a fundamental, inescapable force of nature, not merely a concept.

It humorously emphasizes that gravity isn't just a scientific concept but a universal, non-negotiable rule, a "law" that dictates how things fall, as explored in physics. It's used to celebrate science and physics, often by those who appreciate clever takes on Newton's discoveries and fundamental physical laws.



Newton's law of gravitation

Newton's law does not describe why gravity exists or how it works. Instead, his law describes the magnitude of gravity in terms of gravitational force with the famous inverse square law.

In its classical form, the law shows force that exists between two masses in terms of their distance apart and the gravitational constant. The masses are "point" masses, so their geometry is ignored and the distance between them is the distance from point to point.

$$F = \frac{Gm_1m_2}{r^2}$$

F is magnitude of gravitational force. m1 and m2 are masses. r is the distance between the two masses. G is the gravitational constant $6.67 \times 10^{-11}\ \mathrm{Nm^2kg^{-2}}$

Newton's law of gravitation states that every mass attracts every other mass with a force proportional to their masses and inversely proportional to the square of the distance.

Exercise

What is the magnitude of gravitational force between two touching billiard balls?
Given:
Centers are 7.5×10^{-2} m apart
Mass of each ball is 0.225 kg

Exercise

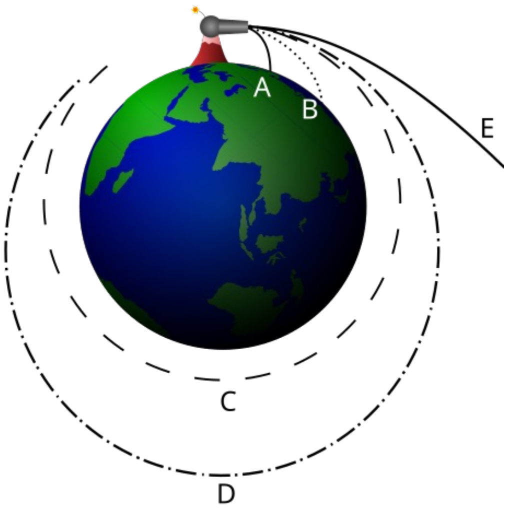
What is the magnitude of gravitational force between two touching billiard balls?
Given:
Centers are 7.5×10^{-2} m apart
Mass of each ball is 0.225 kg

$$
\begin{aligned}
F &= \frac{6.67 \times 10^{-11}(0.225)^2}{(7.5 \times 10^{-2})^2} \\
&\approx 6 \times 10^{-10}\,\mathrm{N}
\end{aligned}
$$

Newton's cannonball

Near Earth's surface, the gravitational force produces an approximately constant acceleration: what we call g.

Newton imagined firing a cannonball horizontally from a very high mountain (Newton's cannonball thought experiment). If the cannonball is slow, it falls to Earth. If faster, it falls but misses the surface; if fast enough, it continuously falls around Earth, this is an orbit. In all cases, the motion is governed by the same gravitational force pulling the object toward Earth's center.

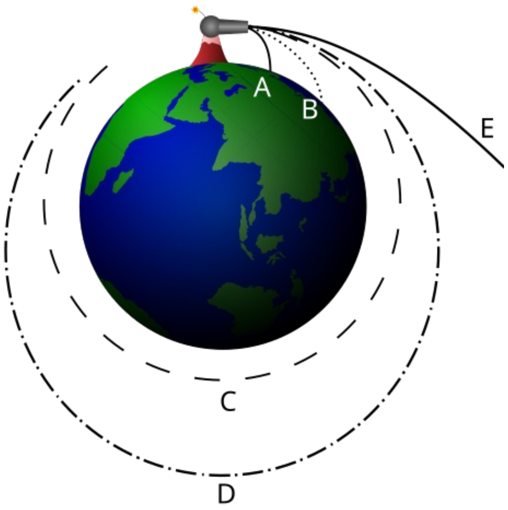


Newton's cannonball and second law of motion

The cannonball’s curved trajectory shows that gravity causes a change in velocity, not just downward motion. This change in velocity is acceleration, produced by a force. From this, Newton formalized the idea that force causes acceleration. This leads directly to Newton’s Second Law of Motion:

$$F = m a$$

Gravitational force is therefore a specific case of a general rule linking force and motion.



By combining the universal law of gravitation with Newton's second law of motion, the acceleration of m_2 due to its attraction by m_1 is:

$$
\begin{aligned}
F &= m_2 a \\
&= \frac{G m_1 m_2}{r^2} \\
a &= \frac{G m_1}{r^2}
\end{aligned}
$$

Gravitational acceleration on Earth

For a bulk (homogeneous, spherical) Earth model, gravity above the surface (R>RE):

$$g = \frac{GM_E}{R^2}$$

g = is gravitational acceleration.

G = is the gravitational constant.

M_E = is the mass of the Earth.

R = is the distance from the center of the Earth.

RE is the radius of the Earth.

Average acceleration at the surface of the Earth is 9.8 m s^{-2}

Exercise: gravitational acceleration Earth - Moon

What is the gravitational acceleration at the surface of the Earth due to the Moon when it is directly overhead (mass of moon, $M_M = 7.3 \times 10^{22}$ kg. approximate distance to moon, $R_M = 3.8 \times 10^8$ m)?

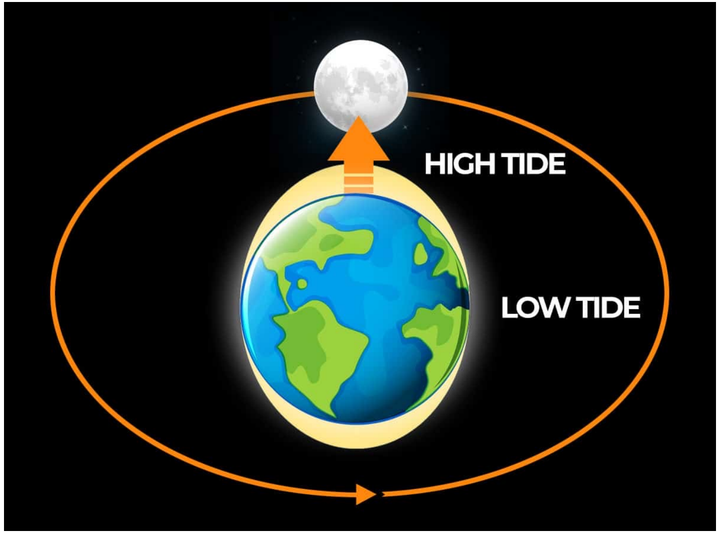
Exercise: gravitational acceleration Earth - Moon

What is the gravitational acceleration at the surface of the Earth due to the Moon when it is directly overhead (mass of moon, $M_M = 7.3 \times 10^{22}$ kg. approximate distance to moon, $R_M = 3.8 \times 10^8$ m)?

$$
\begin{aligned}
g &= \frac{(6.67 \times 10^{-11})(M_M)}{R_M^2} \\
&= 0.000033720\text{ms}^{-2} = 3.4\text{mGal}
\end{aligned}
$$

Results? Tides!

Tides are the regular, periodic rise and fall of sea levels caused primarily by the gravitational pull of the Moon, with additional influence from the Sun. As the Moon orbits Earth, its gravity pulls on the oceans, creating a "bulge" of water (high tide) on the side facing it and a corresponding bulge on the opposite side, while low tides occur in between

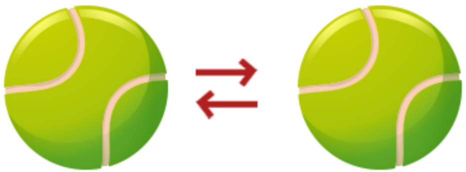


Gravitation vs. gravity:

So far, when we talk about gravity, we often implicitly mean Newton's law of gravitation. Gravitation is a fundamental force: it describes how two masses attract each other. In its simplest form, it tells us the force between two point masses separated by a distance.

What we actually measure in geophysics, however, is not gravitation in this ideal sense. We measure gravity, which is an acceleration. Gravity is the result of gravitation plus additional effects that arise because we are standing on a rotating, non-spherical planet.

Gravitation



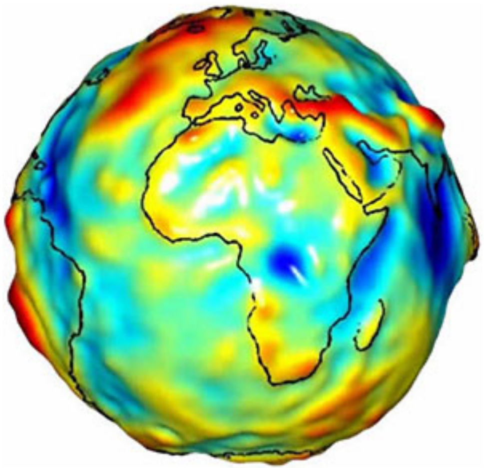
Gravity

Key message: Gravitation tells us how mass creates attraction, gravity is the acceleration we observe on Earth.

What do we measure when we measure gravity?

When a gravimeter measures gravity at Earth's surface, it is not measuring a single effect. The measured gravity acceleration is the combined result of three contributions.

- First, gravity reflects Earth's mass distribution. Variations in density inside the Earth, such as mountains, sedimentary basins, crustal roots, or mantle structure.
- Second, Earth is rotating. Because of rotation, there is a centrifugal acceleration that slightly reduces the effective gravity we feel, especially near the equator.
- Third, Earth is not a perfect sphere. It is an oblate spheroid, flattened at the poles and wider at the equator. This changes the distance to Earth's center and therefore the gravitational acceleration.



What do we measure when we measure gravity?

When a gravimeter measures gravity at Earth's surface, the measured gravitational acceleration has a typical textbook value:

$$g = 9.81 \text{ m/s}^2$$

Actual range on Earth:

$g \approx 9.78 \text{ m/s}^2$ (equator)

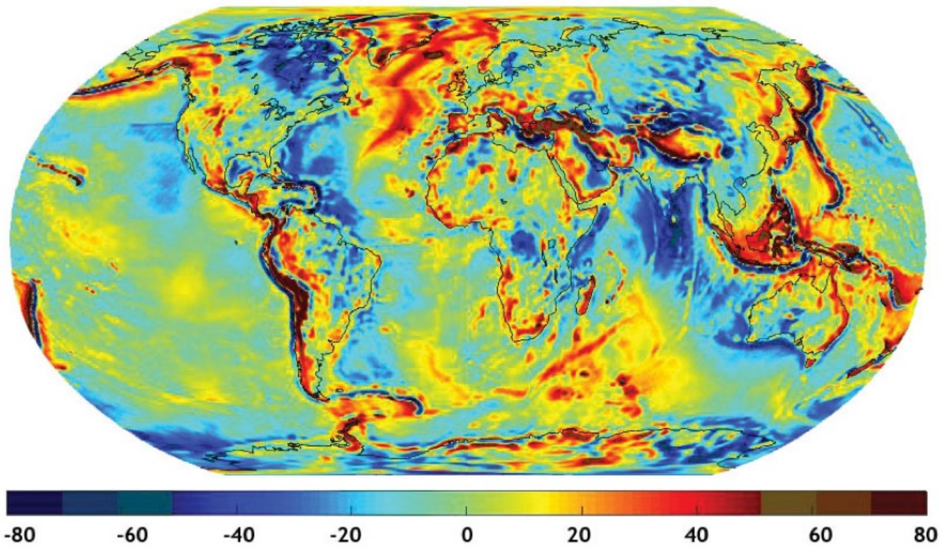
$g \approx 9.83 \text{ m/s}^2$ (poles)

why? centrifugal acceleration acting outward, opposite to gravity

Key point: Gravity is not exactly 9.81 everywhere because Earth rotates, is flattened, and has a heterogeneous interior.

What does gravity measure at large scale?

At the scale of the whole Earth, gravity is not telling us about individual rocks or shallow structures. *Gravity is an integrating measurement*: every point on the surface feels the combined pull of all the mass beneath it, from the crust down to the deep mantle.



Gravity anomalies from ten years (2003-2013) of data in milligal.

1 milligal (or mGal) = 0.00001 m/s^2, which can be compared to the total gravity on the Earth's surface of approximately 9.8 m/s^2. Thus, a milligal is about 1 millionth of the standard acceleration on the Earth's surface.

What does gravity measure at large scale?

At the scale of the whole Earth, gravity is not telling us about individual rocks or shallow structures. *Gravity is an integrating measurement*: every point on the surface feels the combined pull of all the mass beneath it, from the crust down to the deep mantle.

This means gravity is fundamentally sensitive to how mass is distributed laterally and vertically inside the Earth. If mass is redistributed, by thickened crust, by a sinking slab, by hot low-density mantle, gravity changes.

Importantly, gravity does not care about temperature directly, or stress directly. It only cares about density, but density is controlled by all of those processes.

Question:

If two regions have the same surface elevation, could they still produce different gravity signals?

What is the gravity at the centre of the Earth?

At the Earth's surface, gravity is about 9.81 m/s^2, but this value does not stay constant as we move inside the Earth.

What's your guess?

As we go downward from the surface toward the center, gravity first decreases, and it eventually becomes zero at the centre of the Earth.

Why? At the surface, all of the Earth's mass lies beneath you, pulling you inward. As you move inside the Earth, the mass that lies above your position no longer contributes to the gravitational force. Only the mass enclosed within your radius matters. As you go deeper, that enclosed mass decreases, so the gravitational pull weakens.

$$g(r) = \frac{G\,M(r)}{r^2}$$

$r \rightarrow 0 \qquad M(r) \rightarrow 0$

How do we actually measure gravity?

Recap: *Gravity itself is not measured directly as a force. What we actually measure is acceleration. Every gravity measurement is, at its core, an attempt to measure how fast something accelerates when it is allowed to respond to Earth's gravitational field.*

Key conceptual point:

- Gravity measurements are local (they depend on where you are).
- They are relative (we compare tiny differences).
- They are extremely sensitive (changes of one part in a billion matter).

How do we actually measure gravity?

The basic principle: a test mass in free fall.

(physical principle behind all gravimeters)

A gravimeter is an instrument that contains a small test mass whose motion is controlled and observed. Gravity tries to accelerate that mass downward. The instrument measures how strongly gravity pulls on it.

Gravimeter

How do we actually measure gravity?

Method 1: Absolute gravimeters

In an absolute gravimeter, the idea is brutally simple and comes straight from Newton.

You take a small mass and let it fall freely inside a vacuum chamber.
You track its position as a function of time using a laser interferometer.
From the trajectory z(t), you compute the acceleration.

No calibration, no reference location, no comparison. You are literally measuring acceleration from first principles.

Gravimeter

How do we actually measure gravity?

Method 1: Absolute gravimeters

Why is this difficult? Because:

- the drop distance is small (tens of cm)
- air drag must be eliminated (vacuum)
- vibrations must be isolated
- timing and distance must be measured with extreme precision.

But conceptually, this is the cleanest measurement of gravity you can do.

Gravimeter

How do we actually measure gravity?

Method 2: Relative gravimeters

Gravimeter

Most gravimeters used in practice are relative gravimeters. They do not drop a mass. Instead, they hold a mass in place and measure how much gravity tries to move it. The classic setup is a mass attached to a spring.

Gravity pulls the mass downward. The spring resists that motion. Gravity is inferred from how much the spring stretches under the weight of the mass.

How do we actually measure gravity?

$$F_g = mg$$

Gravity pulls the mass downward

$$F_s = k\,\Delta z$$

Spring resists (Hooke's law)

$$F_s = F_g$$

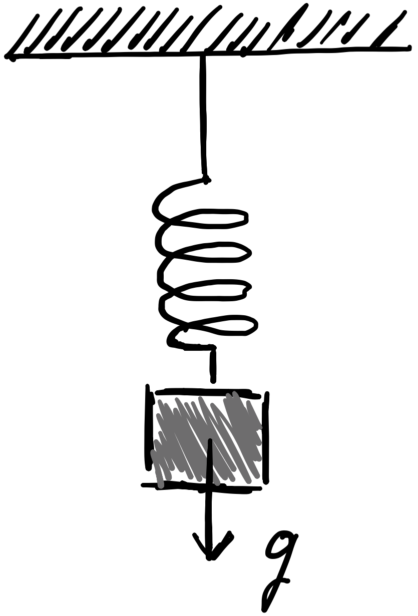
At the equilibrium

$$k\,\Delta x = mg$$

$$g = \frac{k}{m}\Delta z$$

Elasticity 🧐

Hooke's Law (lecture 2)



Gravity measurements are very sensitive!

Typical gravity variations due to Earth structure are:

$\Delta g \sim 10^{-6}$ to 10^{-9} g

That means:

- density variations in the mantle
- crustal thickness changes
- post-glacial rebound
- mass redistribution after earthquakes

All produce measurable changes, even though gravity itself feels “constant” to us.

Measuring gravity from ground instruments to satellites

Ground-based measurements are powerful but limited:

- They are sparse.
- They are affected by topography and access.
- They cannot easily capture global patterns.

To truly observe gravity at the scale of the whole Earth, we need to go to space.

GRACE: measuring gravity from space

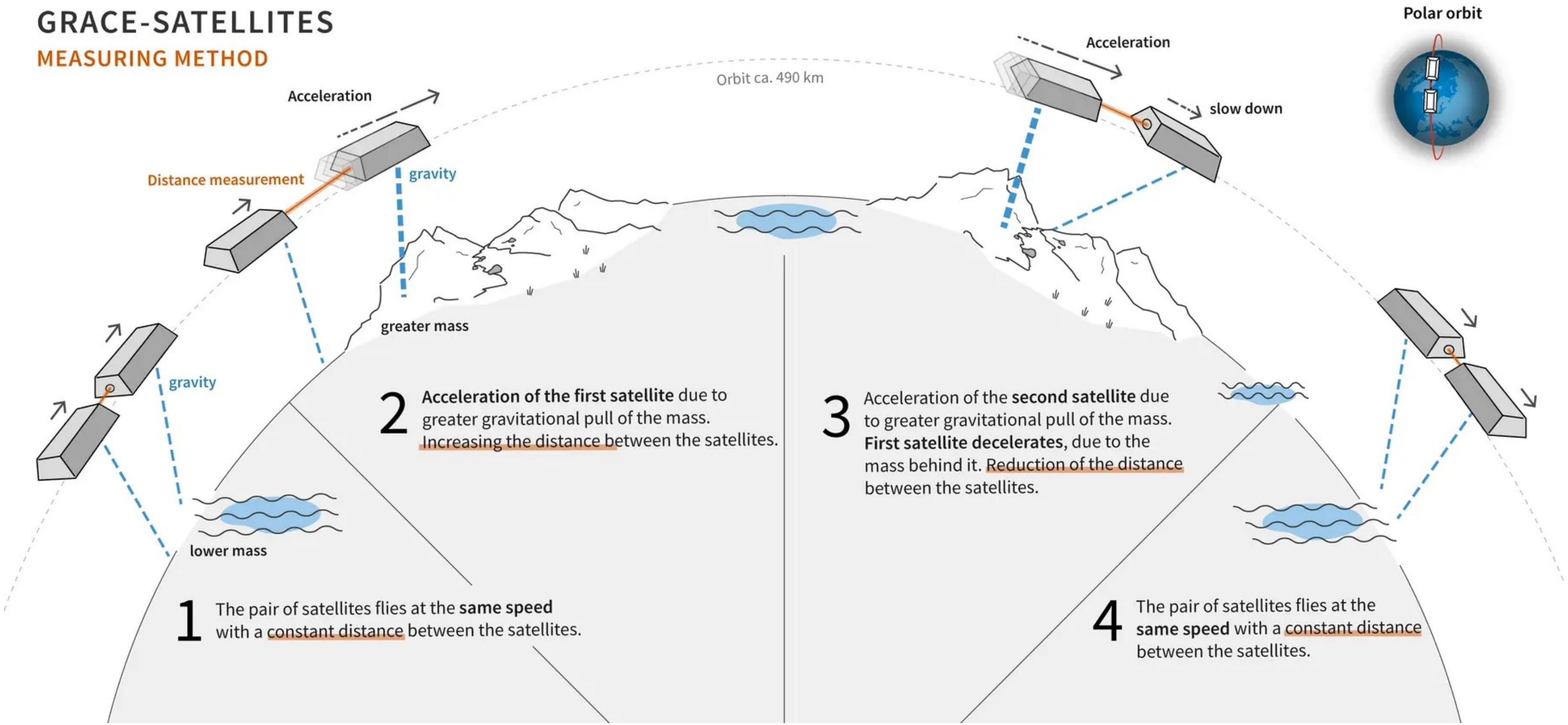
The breakthrough came in 2002 with the launch of GRACE (Gravity Recovery and Climate Experiment), a joint NASA–DLR mission. GRACE introduced a fundamentally new way of measuring gravity: it measured changes in distance between two satellites flying in tandem. This innovation allowed gravity variations to be observed directly and continuously at the global scale.

GRACE: how it works

1. Two satellites orbit Earth in the same path, separated by ~200 km.
2. Gravity is not uniform, so their accelerations are not identical.
3. Variations in gravity stretch or compress the distance between them.
4. A microwave (and later laser) ranging system measures distance changes with micrometer precision.
5. From these distance changes, we invert for Earth’s gravity field.

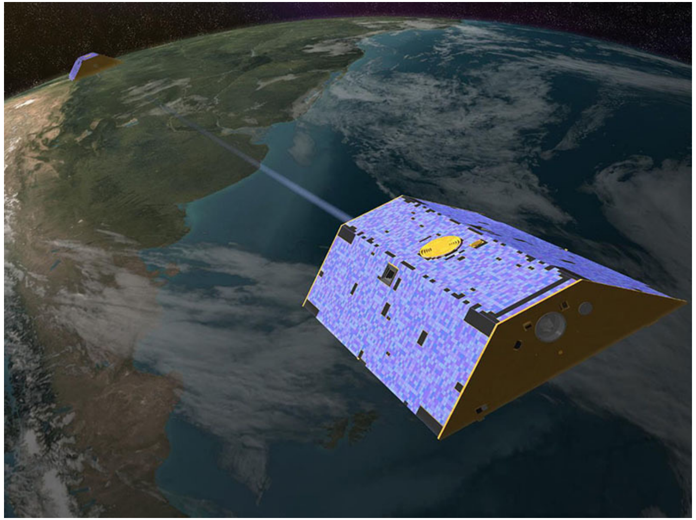
GRACE does not “see” mountains or slabs directly. It sees how mass redistributes itself through its gravitational pull.

GRACE: how it works



GRACE: measuring gravity from space

What made GRACE revolutionary was not just its accuracy, but the fact that it measured time-variable gravity. For the first time, gravity was no longer treated as a static property of the Earth. GRACE revealed that Earth's gravity field changes over months to years as mass moves within the Earth system. This transformed gravity from a purely geometric field into a dynamic observable.



Today's GRACE models represent one of the most precise large-scale datasets in geophysics. They provide a direct link between gravity observations and solid Earth structure, allowing us to study density variations in the lithosphere and mantle, long-wavelength geodynamic processes, and the ongoing redistribution of mass within the Earth.