Rheology

Rheology

Rheology is the science of how matter deforms and flows under applied forces, studying the relationship between stress (force) and strain (deformation) in materials like liquids, gels, and solids, focusing on properties such as viscosity, elasticity, plastic flow.

What do we call rheology?

1) LOGOS (study) of RHEO (flow)

= the science that deals with the deformation and flow of materials, both solids and liquids.

2) Now rheology is also used to name “the response of material to an applied force”

- Experimentalists produce rheological parameterisations

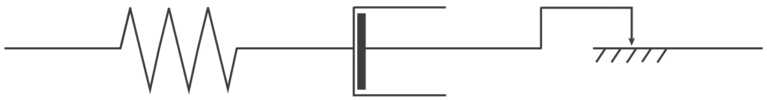
- Thermo-mechanical modelers use rheology as an input to predict plate and mantle/rock dynamics

Rock rheology

According to stress, strain rates and Pressure-Temperature (PT) conditions, rocks can experience 3 types of deformation:

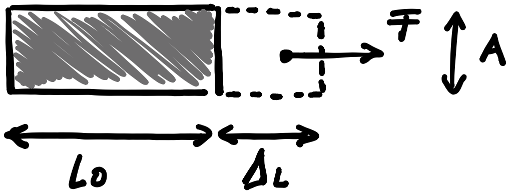
ELASTIC + VISCOUS + BRITTLE/PLASTIC

Maxwell visco-elasto-plasticity (V-E-P)



In continuum mechanics, rheology (the study of material flow – deformation) is used to characterize the mechanical behavior of visco-elasto-plastic media.

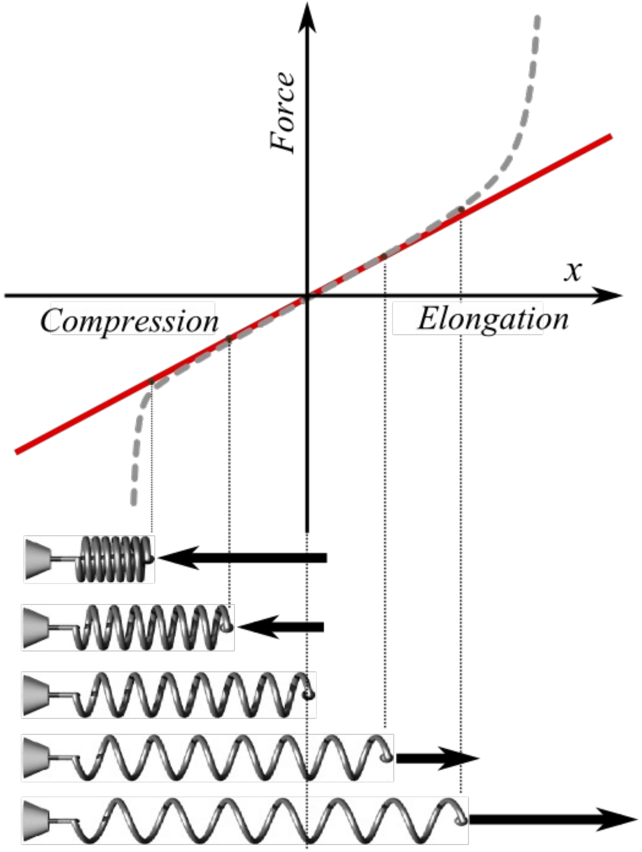
Elasticity



$\sigma = F/A$ Normal stress

$\varepsilon = \Delta L / L_0$ Normal strain

Pole in bamboo, aluminium, fiberglass, carbon fiber



Elasticity

$$E = \frac{Pa}{\text{dimensionless}} = Pa$$

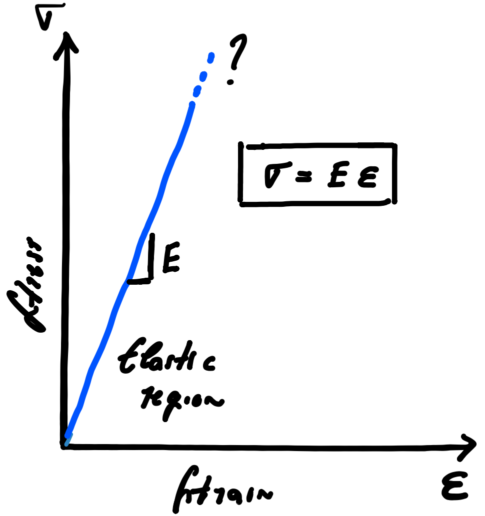
Soft rubber $= 10^6$ Pa

Steel / Rocks $= 10^{10} - 10^{11}$ Pa

Hooke's Law in 1D

In the elastic regime, many solids obey a linear relationship between stress and strain. ***E*** **is Young's modulus** (stiffness).

- Large E: material is stiff, small strain for a given stress.
- Small E: material is compliant, large strain for a given stress.



Physically: E measures how much stress is needed to produce a given strain.

Elasticity in rocks – crystal bonding

When stress is applied to a rock: forces between atoms shift slightly, bonding geometry changes a tiny bit, the crystal adjusts its shape. **If the deformation is small, the change in shape (strain) is proportional to the applied stress**. This is exactly the same idea as a spring obeying Hooke's law:

Stress is proportional to strain

The proportionality constant (like the "spring stiffness") depends on the strength of the atomic forces. In rocks, these stiffnesses are the elastic moduli (e.g., Young's modulus, shear modulus).

When does elasticity matter in geophysics?

Elastic behavior dominates when:

- Strains (deformation) are very small
- Stresses are below the “yield limit”
- Deformation happens quickly (high strain rate)
- Atoms cannot diffuse or rearrange

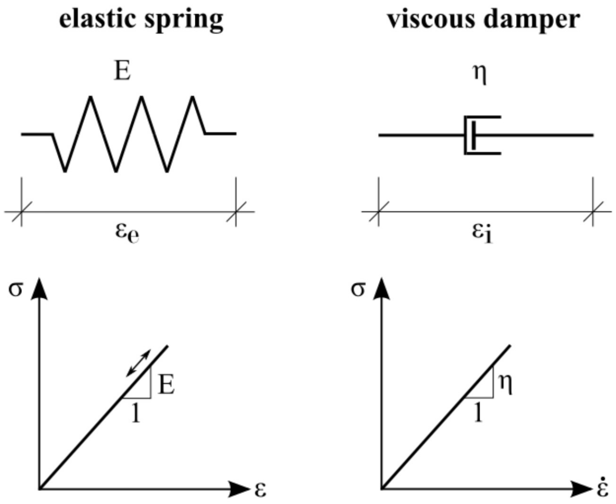
This is exactly the regime of:

- Seismic waves (nanostrain, millisecond timescales)
- Small bending or flexure at short timescales
- Instantaneous response to loads (before viscous relaxation)

Viscous rheology

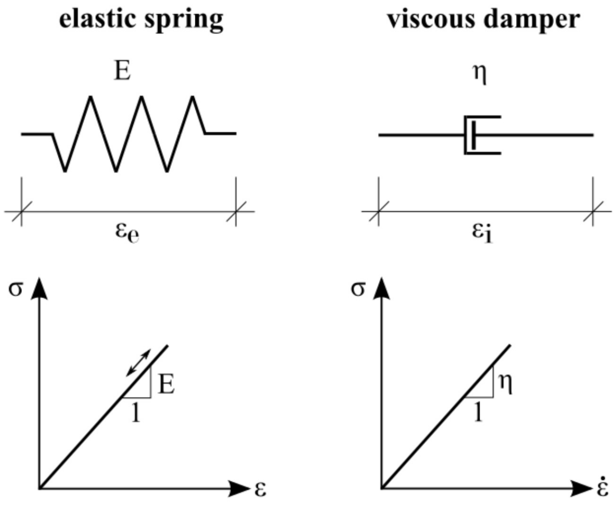
- Viscous deformation = strain proportional to **stress acting over time**.
- Unlike elasticity (instantaneous response), viscosity depends on **strain rate**.
- Rocks deform viscously when atoms can **diffuse, climb, slide** on geological timescales.
- Dominant at high T, long timescales, deep crust/mantle.

In the elastic regime, deformation is instantaneous and reversible. In the viscous regime, deformation accumulates with time and is permanent.



Newtonian Viscous Flow (basic law)

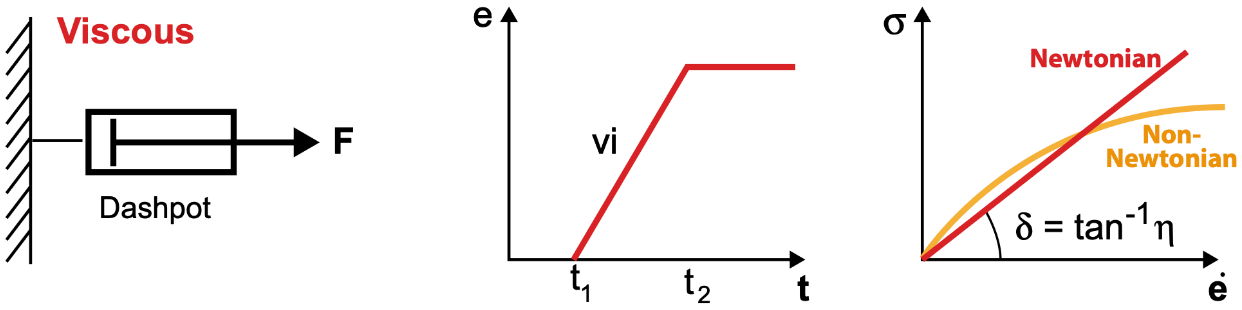
- Newtonian viscosity: $\sigma = \eta \, \dot{\varepsilon}$
- Linear relation between stress and strain rate.
- Constant viscosity.
- Good approximation for diffusion creep or fluid-like behavior.



Non-Newtonian flow (power-law creep)

- Dislocation creep dominates in most mantle and warm crustal rocks.

- Stress-strain rate relation: $\dot{\varepsilon} = A\sigma^n \exp\left(-\frac{Q}{RT}\right)$



- n > 1: strain rate highly sensitive to stress (2–5 strongly nonlinear)
- A: material constant / Q: activation energy / T: temperature / R: Gas constast
- Controls lithospheric strength at long timescales.

Arrhenius term

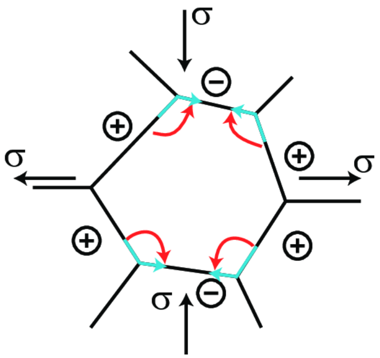
$$\dot{\varepsilon} = A\sigma^n \exp \boxed{\left(-\frac{Q}{RT}\right)}$$

The Arrhenius term is in fact it is extremely common in physics and chemistry. Comes from **thermal activation theory**. It appears whenever a process requires energy to overcome a barrier. Here, that barrier is the energy needed for atoms or defects in a crystal lattice to move. The parameter Q is called the activation energy. It represents how difficult it is for dislocations to move through the mineral structure. Minerals with strong bonds have higher Q and are harder to deform. The constant R is the gas constant, and T is absolute temperature.

As temperature increases, the exponential term increases very rapidly. **This means that a small increase in temperature can lead to orders-of-magnitude increase in strain rate.**

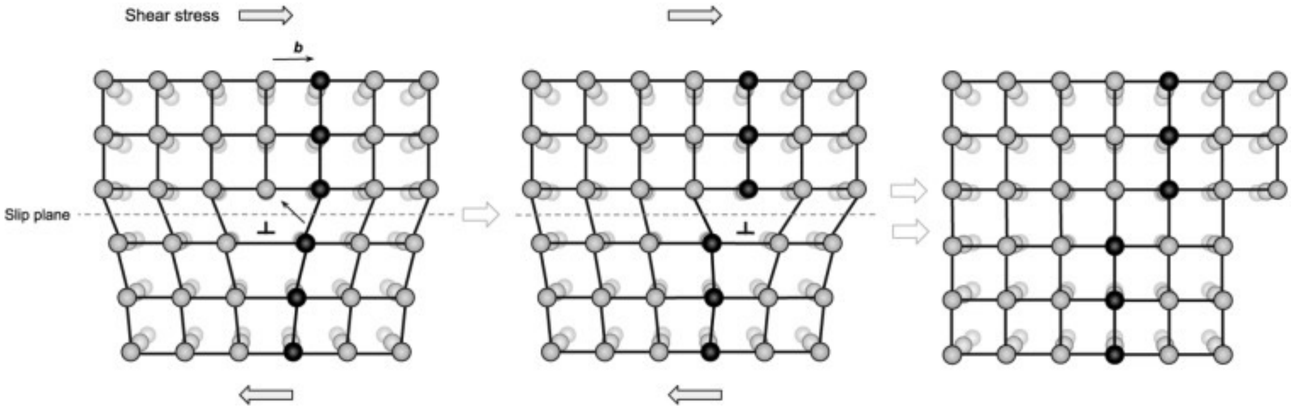
Diffusion vs. Dislocation creep

Diffusion Creep (Newtonian, n = 1): Deformation occurs by atomic diffusion: atoms move through the crystal lattice or along grain boundaries. No internal lattice defects (dislocations) are required to move. Strongly grain-size dependent: fine-grained rocks deform faster. Dominant at low stress, high temperature, small grain size, and often in the lower crust or near solidus conditions.



Diffusion vs. Dislocation creep

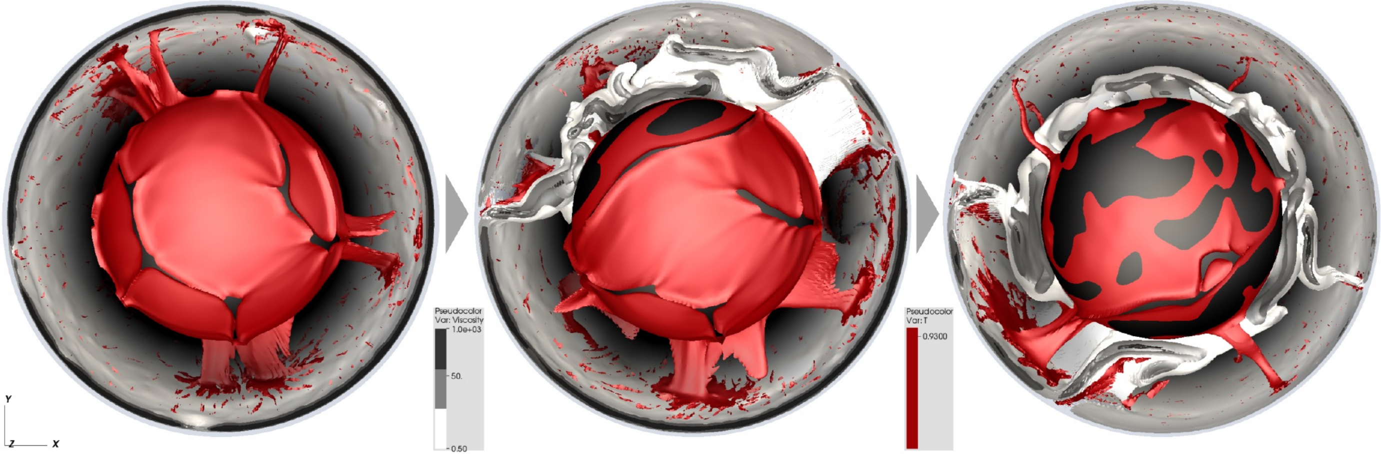
Dislocation Creep (Power-law, n = 3–5): Deformation occurs by movement of dislocations (crystal defects) through the lattice. Requires breaking and reforming bonds along defect lines. Highly stress-sensitive, giving a power-law relation. Grain-size independent (controlled by crystal defects, not diffusion paths). Dominant at higher stress, high temperature, and in upper mantle peridotite and warm lower crust.



Viscosity in Geophysics

Effective viscosity depends on: temperature, stress, grain size, water content, melt. Rocks deform viscously at: mantle conditions, deep crust, long-term plate motions

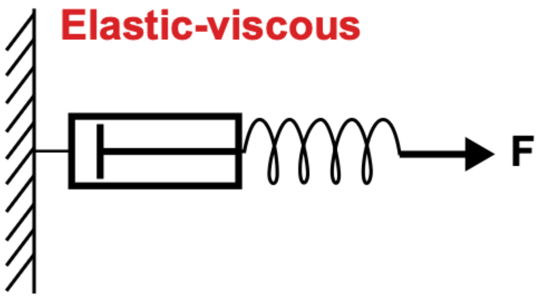
Viscosity can vary by 10+ orders of magnitude across Earth.

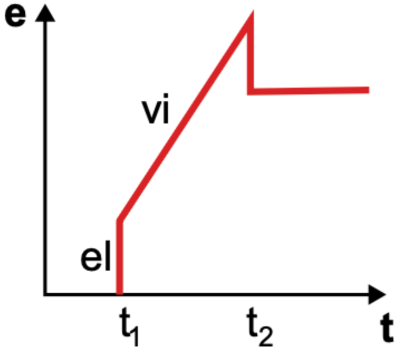


Maxwell Model (Elastic + Viscous)

- Combines spring (elastic) and dashpot (viscous) in series.
- Short timescale → elastic.
- Long timescale → viscous.
- Governs post-seismic relaxation, glacial rebound.

$$\dot{\varepsilon} = \frac{\sigma}{\eta} + \frac{1}{E}\dot{\sigma}$$

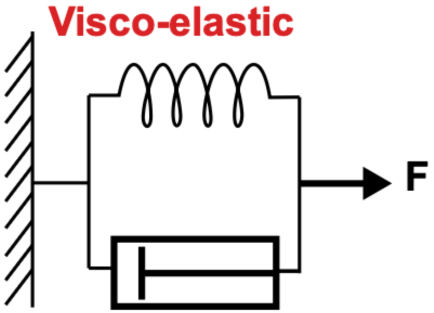


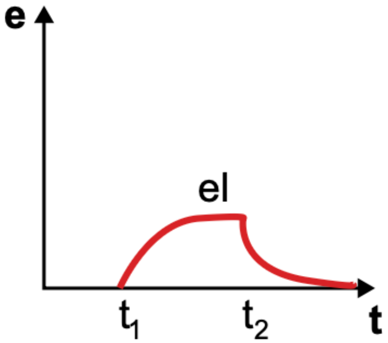


Kelvin–Voigt Model (Viscous + Elastic in parallel)

- Combines a spring (elastic) and a dashpot (viscous) in parallel.
- Under an applied stress, both elements deform simultaneously.
- The total strain is the same in both elements, but each carries part of the stress.
- Predicts time-dependent approach to deformation: slow creep toward a finite elastic limit.
- Good model for anelastic behavior and delayed elasticity, like transient deformation.

$$\sigma = E\,\varepsilon + \eta\,\dot{\varepsilon}$$





Recap: When does viscosity matter?

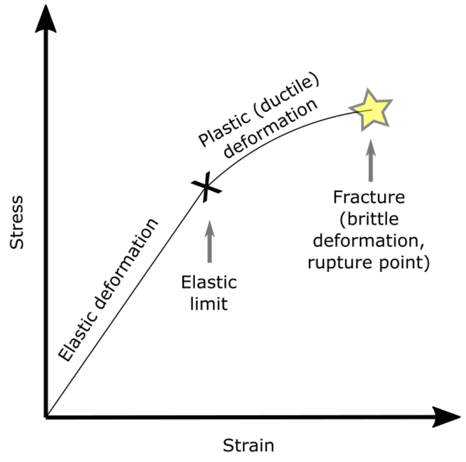
- High temperature
- Long timescales (10^3–10^6 years)
- Low stresses
- Ductile lower crust, upper mantle

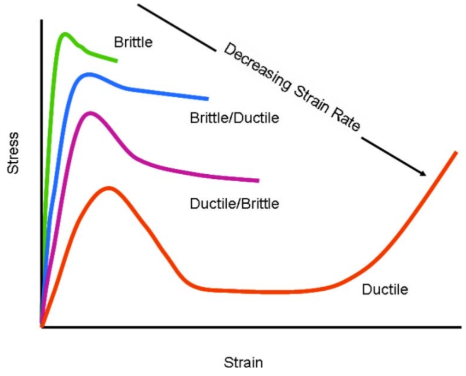
Non-Newtonian fluids

Plastic/brittle rheology

What Is Plastic/Brittle Deformation?

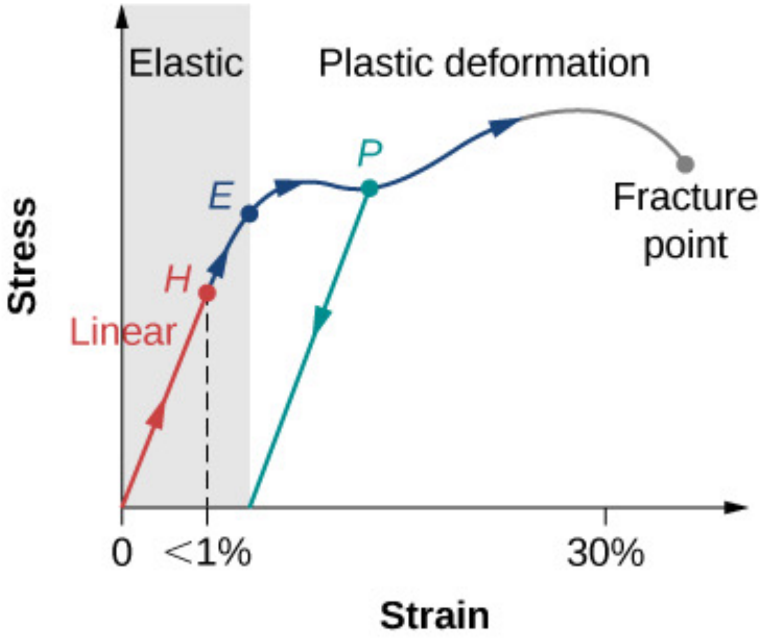
- Plastic = permanent deformation after yield stress is exceeded.
- Brittle failure = sudden loss of strength and frictional slip.
- Elastic → plastic transition: material yields when stress reaches a critical value.
- Depth-dependent in Earth (controlled by pressure, temperature, fluids).





Plastic/brittle rheology

- Initial linear elastic region.
- Yield point
- Beyond yield: plastic flow (permanent strain).
- Failure at peak strength in brittle materials.



Recap: Mohr circle

$$\sigma = \begin{pmatrix} \sigma_{xx} & \sigma_{yx} \\ \sigma_{xy} & \sigma_{yy} \end{pmatrix}$$

$\sigma_n(\theta)$ and $\tau(\theta)$

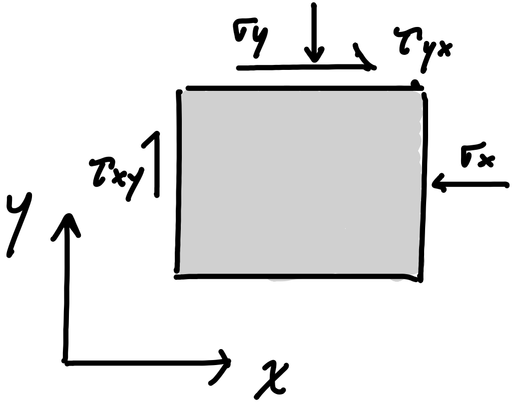
The PRINCIPAL STRESSES occur where

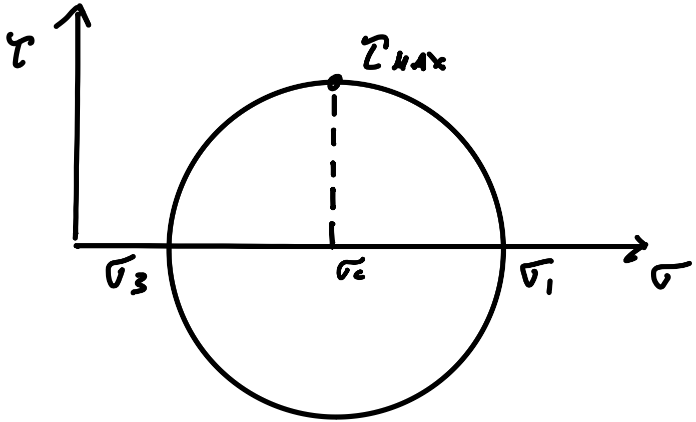
$\tau(\theta) = 0$

$$\sigma_{1,2} = \frac{\sigma_{xx} + \sigma_{yy}}{2} \pm \sqrt{\left(\frac{\sigma_{xx} - \sigma_{yy}}{2}\right)^2 + \sigma_{xy}^2}$$

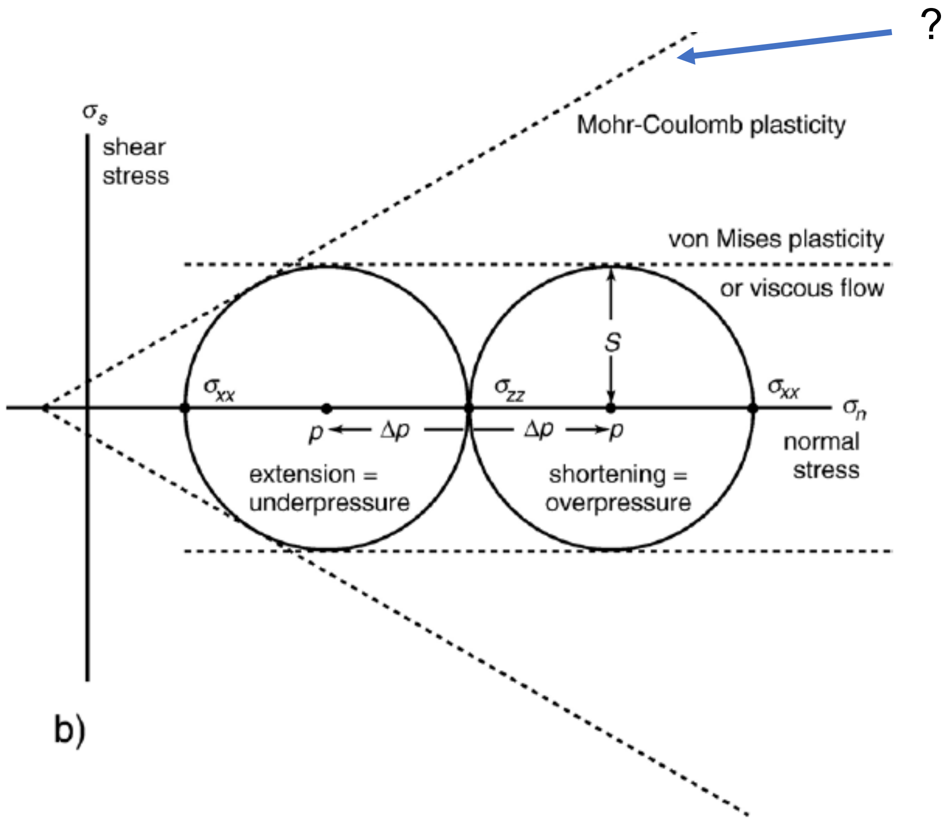
if you know σ_1 and σ_2, you know the entire stress state in 2D.

$$\sigma_c = \frac{\sigma_{xx} + \sigma_{yy}}{2}$$



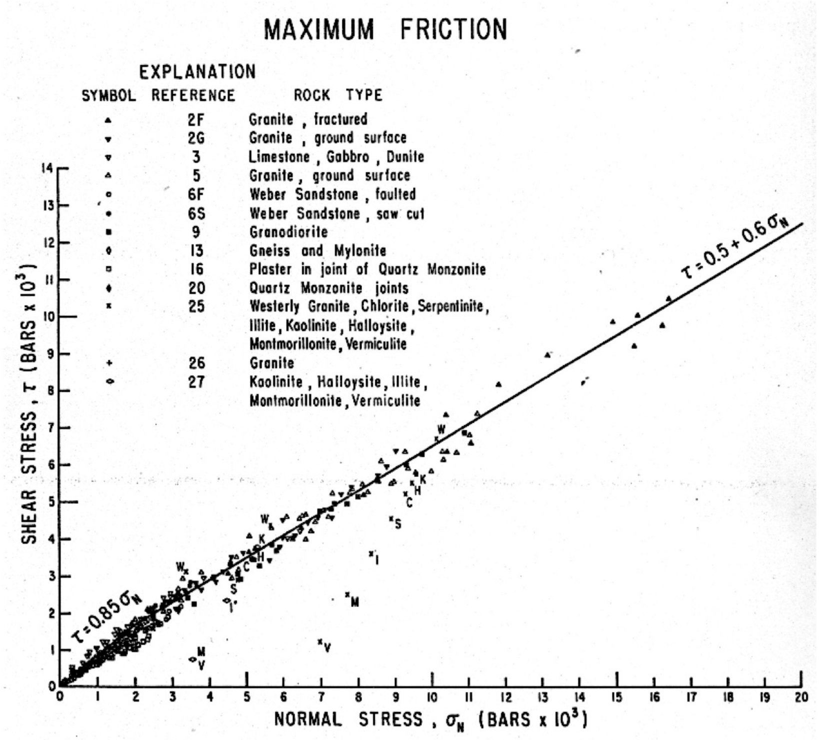


Recap: Mohr circle



Mohr–Coulomb Criterion (general idea)

Byerlee's friction (or Byerlee's Law) describes the relationship between the shear stress (force to slide) and normal stress (force pressing together) for rocks, showing that friction is low at low stress (around 0.85 coefficient) but becomes more constant and lower (around 0.6 coefficient) at the high stresses found in the Earth's crust, explaining how faults slip and earthquakes happen

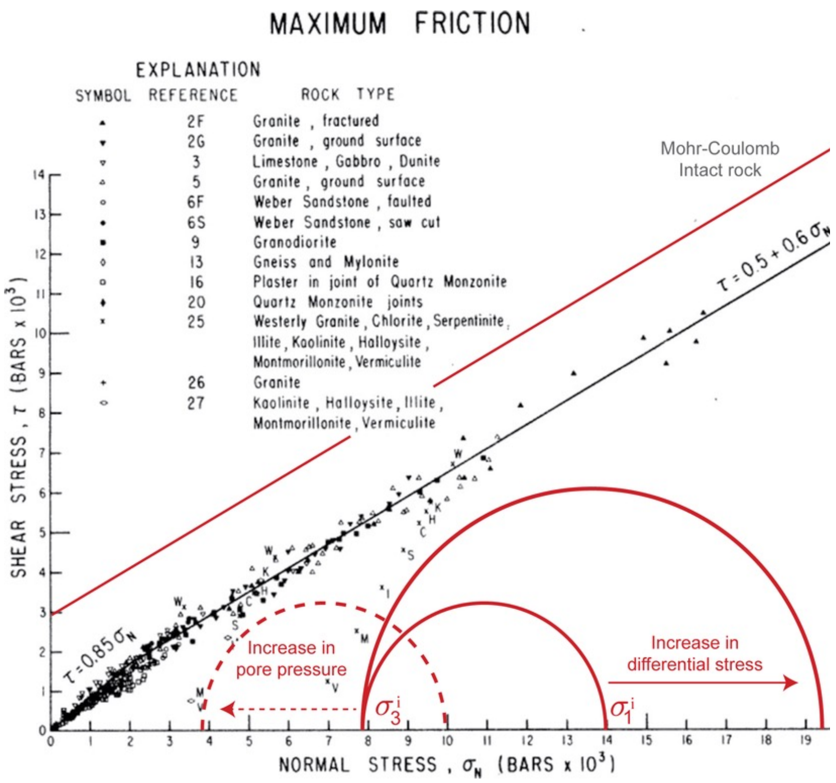


Mohr–Coulomb Criterion (General Idea)

Most rocks fail according to frictional strength:

$$\tau = c + \mu\sigma_n$$

- Strength increases with confining pressure.
- Temperature and pore fluids modify strength.



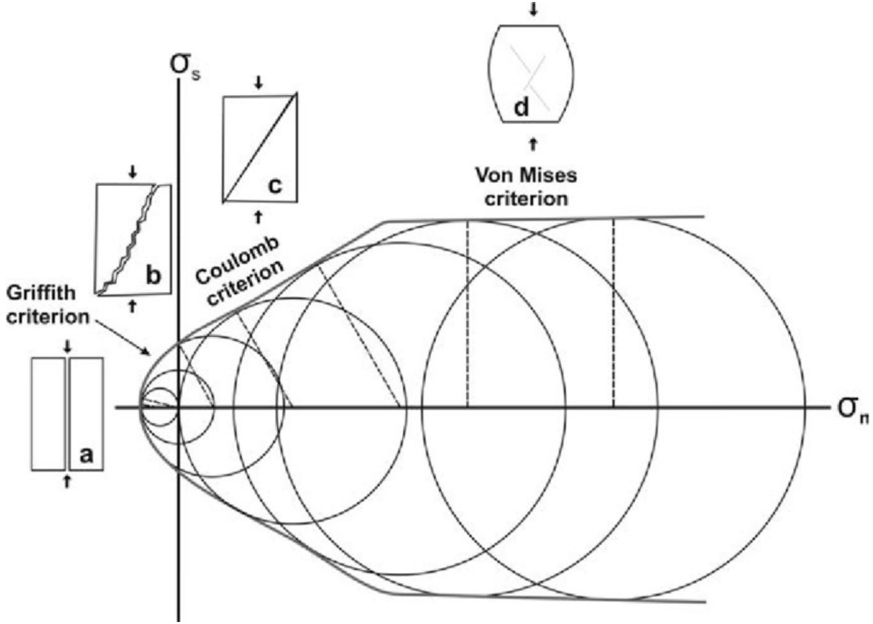
Plastic Flow Laws in Geodynamics

- Plastic flow allows deformation without unlimited stress increase.
- Used in lithosphere models to limit stress to yield criterion.

Common yield laws:

- Drucker–Prager (smooth Mohr–Coulomb)
- Von Mises (pressure-independent)

(They control lithospheric strength.)



Fracture style as a function of confining pressure

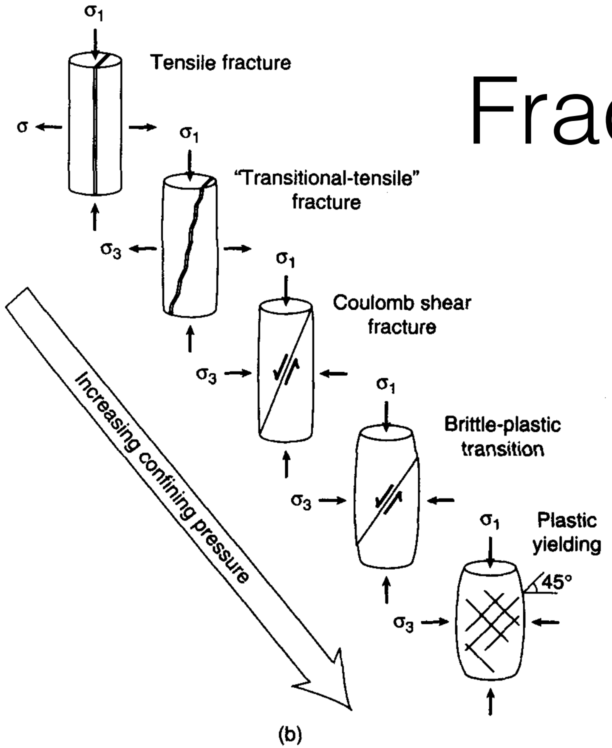
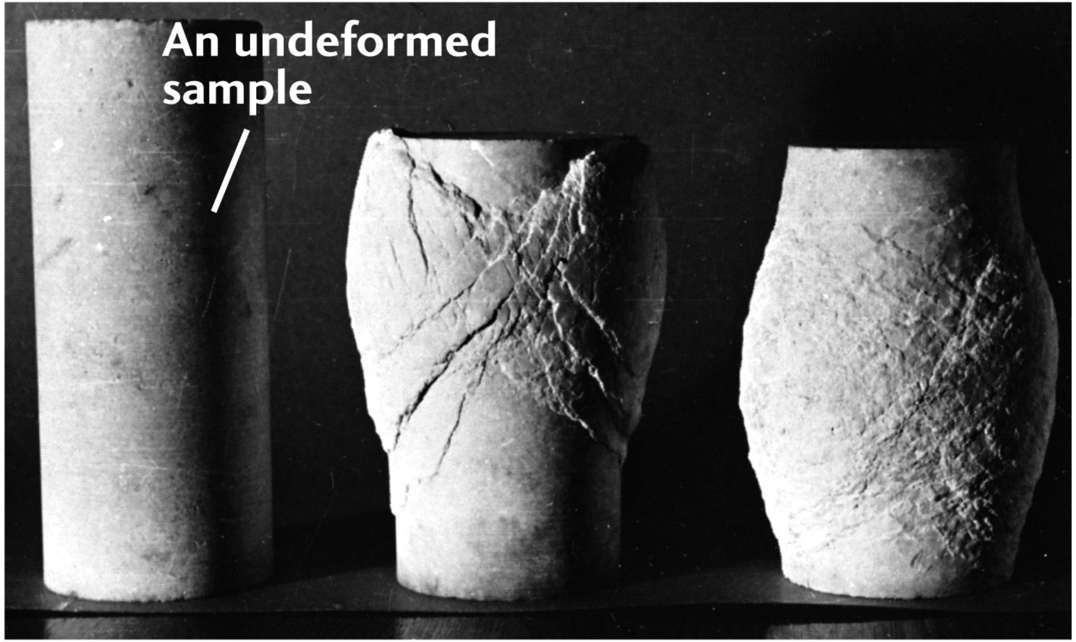


Figure 6.25 (a) A representative composite failure envelope on a Mohr diagram. The different parts of the envelope are labeled, and are discussed in the text. (b) Sketches of the fracture geometry that forms during failure. Note that the geometry depends on the part of the failure envelope that represents failure conditions, because the slope of the envelope is not constant.

Plastic vs. viscous deformation

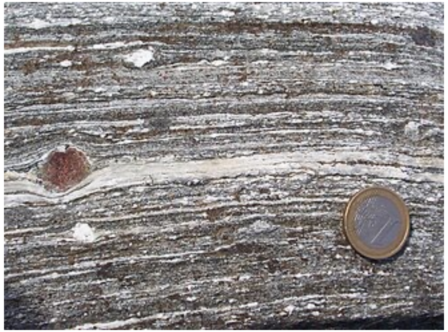


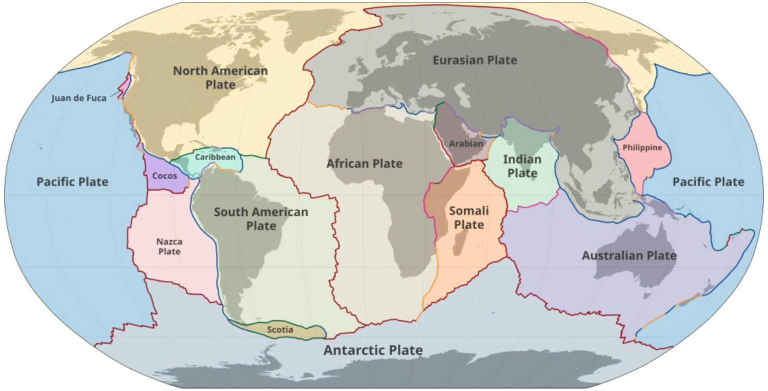
(a) (b) (c)

brittle/plastic viscous

Plasticity in the Lithosphere

- Permanent deformation in faults, shear zones, plate boundaries.
- Plastic yield limits stress in models → prevents unrealistic values.
- Coupled with viscosity → elasto–visco–plastic lithosphere.





From failure criteria to lithospheric strength

RECAP:

- Mohr–Coulomb gives the stress level at which brittle failure occurs for given normal stress.
- Viscous flow laws give the stress required to maintain a given strain rate at depth.
- **Lithosphere strength is the minimum of these two mechanisms at each depth.**
- This leads to the concept of the **yield strength envelope.**

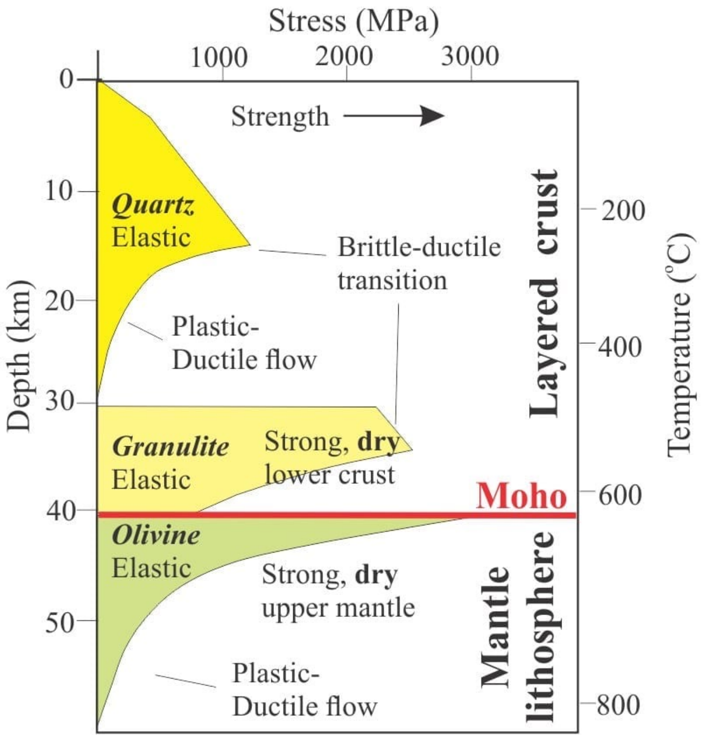
What is a yield strength envelope?

- A Yield Strength Envelope (YSE) shows how the maximum sustainable stress varies with depth.

It combines:

- brittle strength (frictional, pressure-dependent)
- ductile strength (temperature-dependent, viscous creep)

The lithosphere fails whichever mechanism gives lower strength.



Dry lower crust and mantle lithosphere

Brittle Strength with Depth (Mohr–Coulomb)

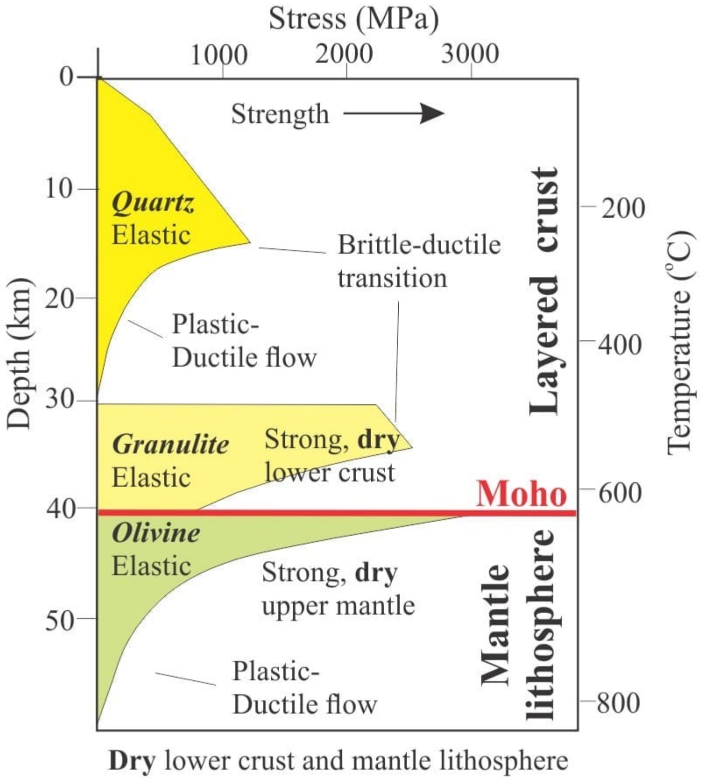
$$\sigma_{\text{brittle}} = c + \mu\,\sigma_n$$

$$\sigma_n(z) = \rho\,g\,z$$

$$\sigma_{\text{brittle}}(z) = c + \mu\,\rho g z$$

Interpretation:

- Brittle strength increases linearly with depth.
- More pressure → stronger rock → harder to break by brittle failure.
- Near surface: very weak (low confinement).
- Deep crust: too strong to fail brittlely.



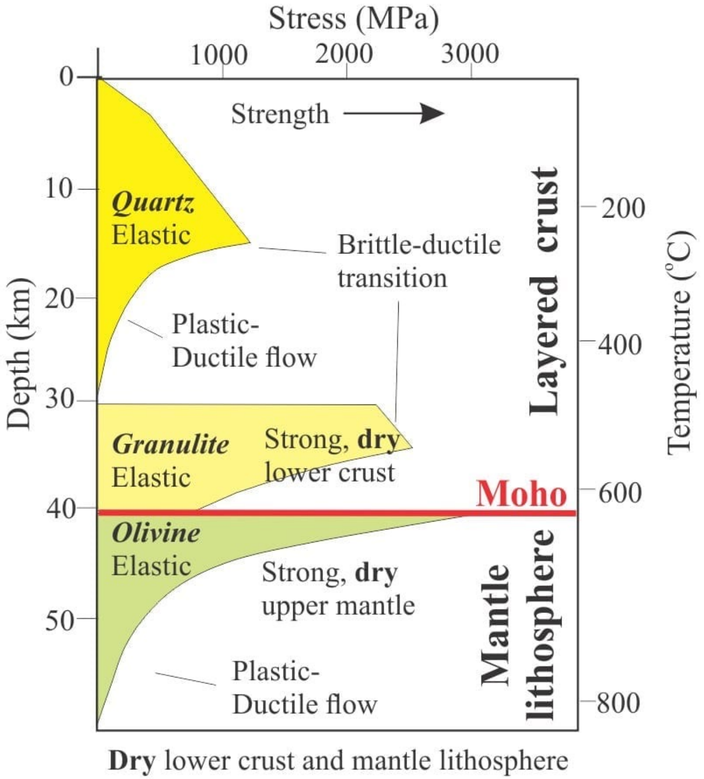
Ductile (Viscous) strength with depth

Viscous creep law: $\dot{\varepsilon} = A\sigma^n \exp(-Q/RT)$

Solve for stress: $\sigma_{\text{ductile}} = \left(\frac{\dot{\varepsilon}}{A} \exp(Q/RT) \right)^{1/n}$

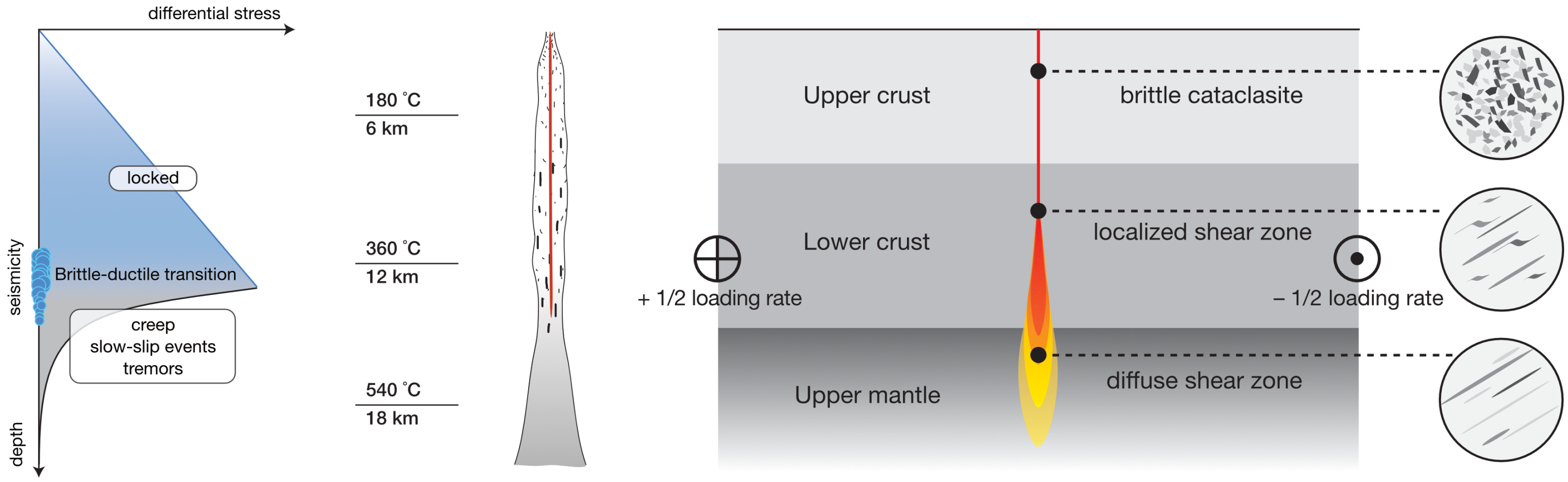
Interpretation:

- Temperature increases with depth → ductile strength decreases with depth.
- Rocks weaken rapidly at high T.
- Deformation becomes dominated by dislocation creep at mid crust/upper mantle temperatures.

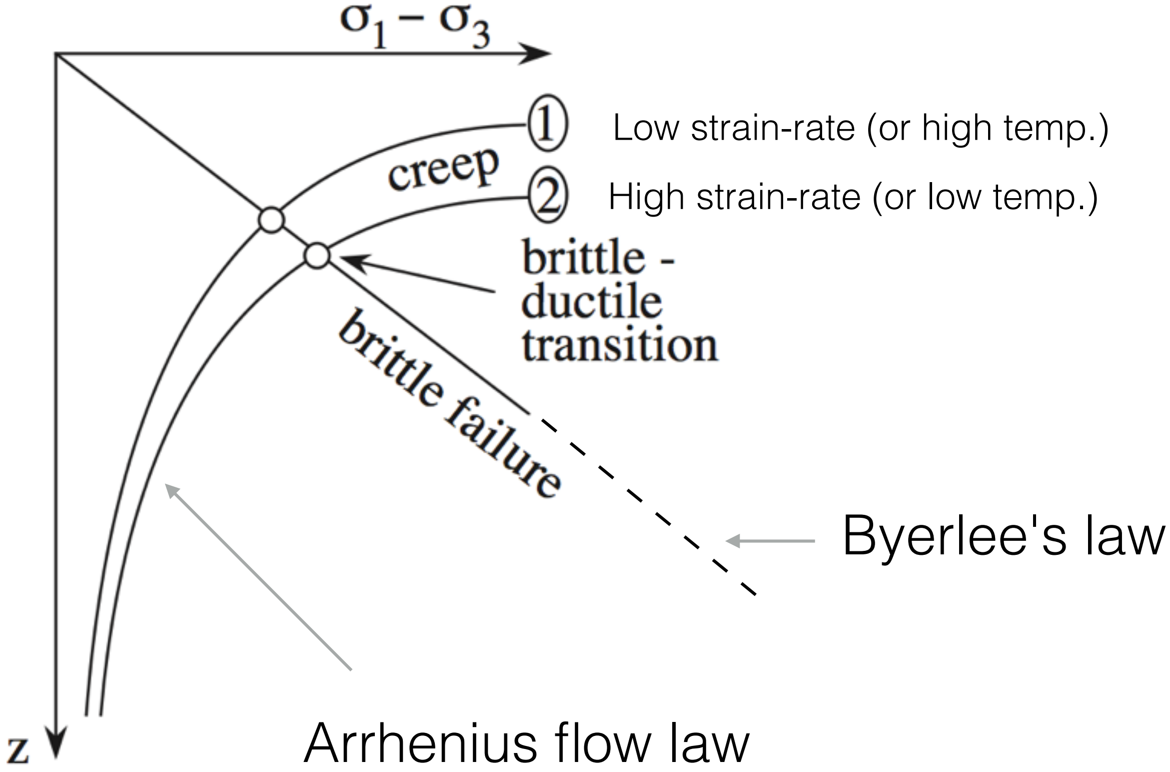


Brittle–Ductile Transition (BDT)

- Near surface: brittle strength increases with depth via confining pressure.
- At depth: viscous strength decreases with temperature.
- Intersection gives BDT.
- Controls earthquake depth distribution.



Brittle–Ductile Transition (BDT): assumptions

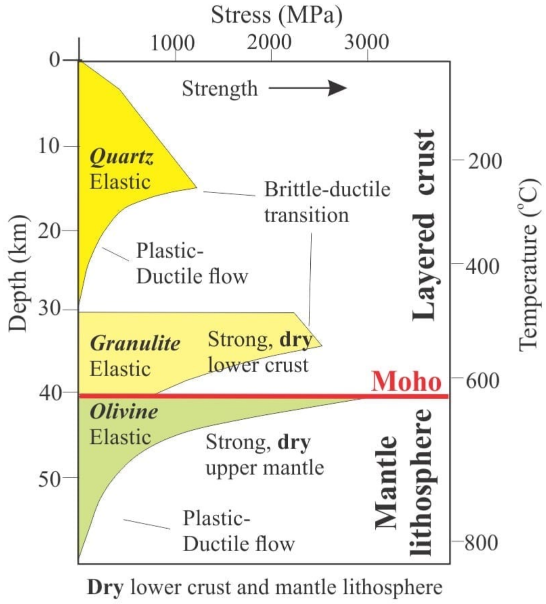


Physical meaning of the Yield Strength Profile (YSP)

YSE shows the maximum differential stress the lithosphere can support at any depth.

Controls:

- lithosphere rigidity
- flexure under loads
- mountain building
- plate boundary strength
- earthquake depth distribution



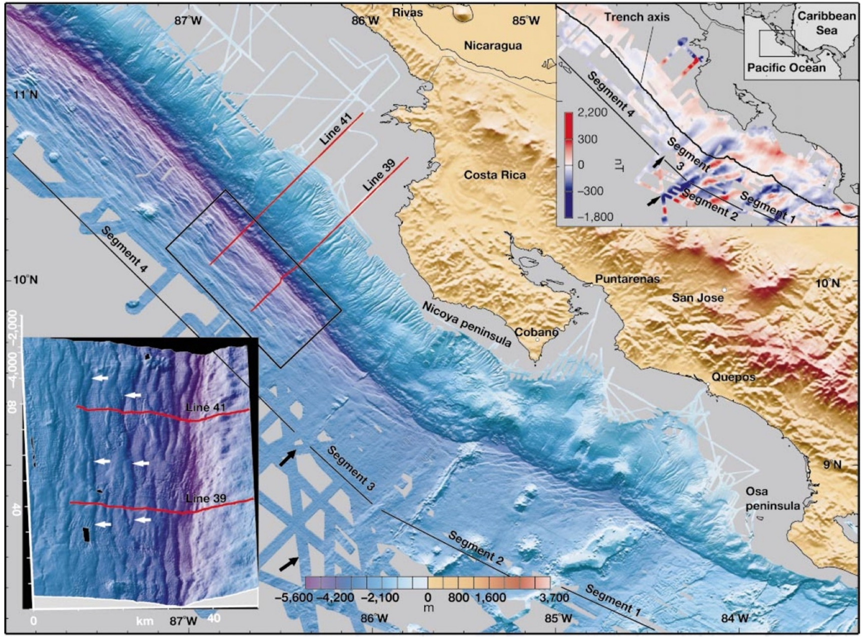
Provides input for geodynamic models (lithosphere strength maps).

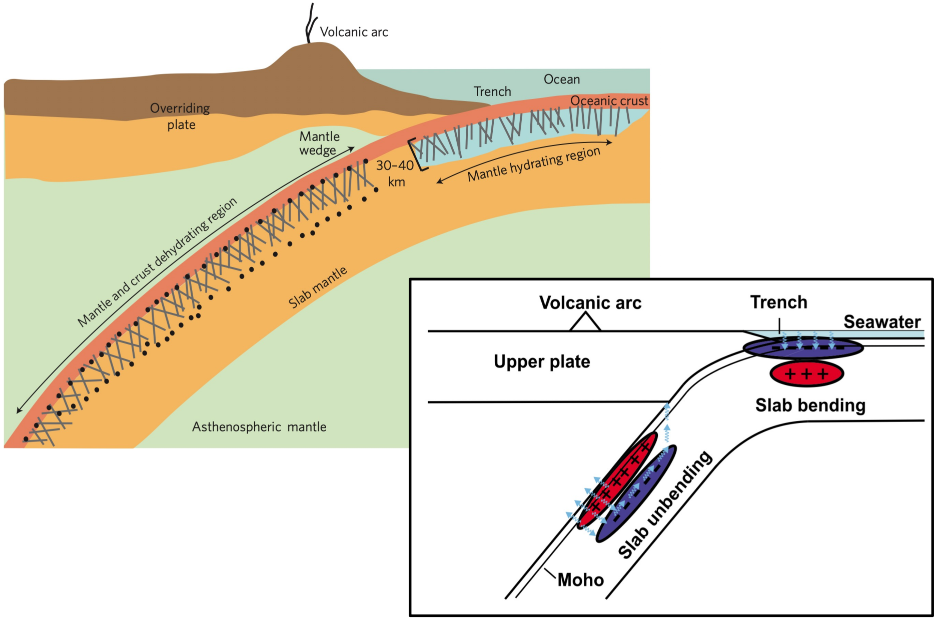
EXAMPLES

Flow in ice = solid water

Elephant foot glacier, Greenland

Geodynamic examples: bending of oceanic plates

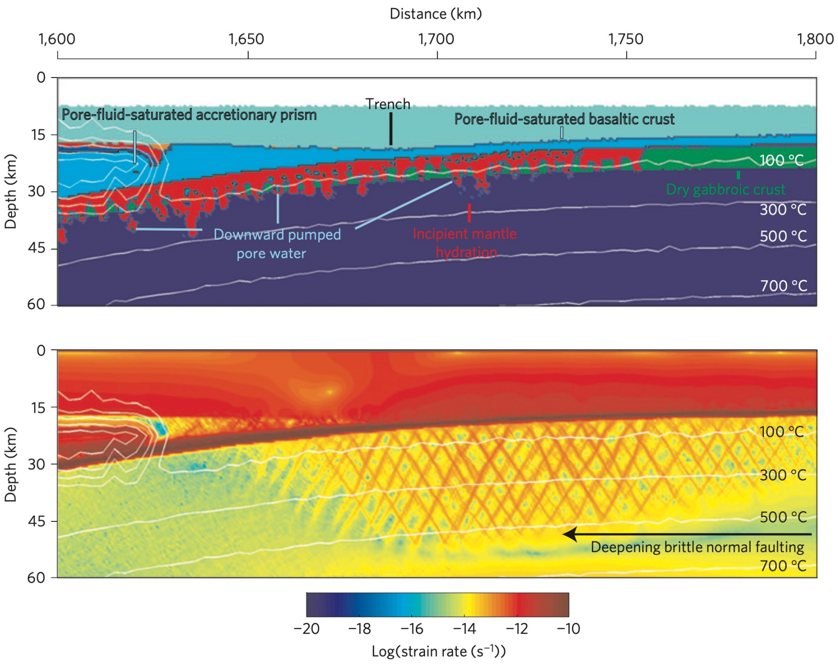


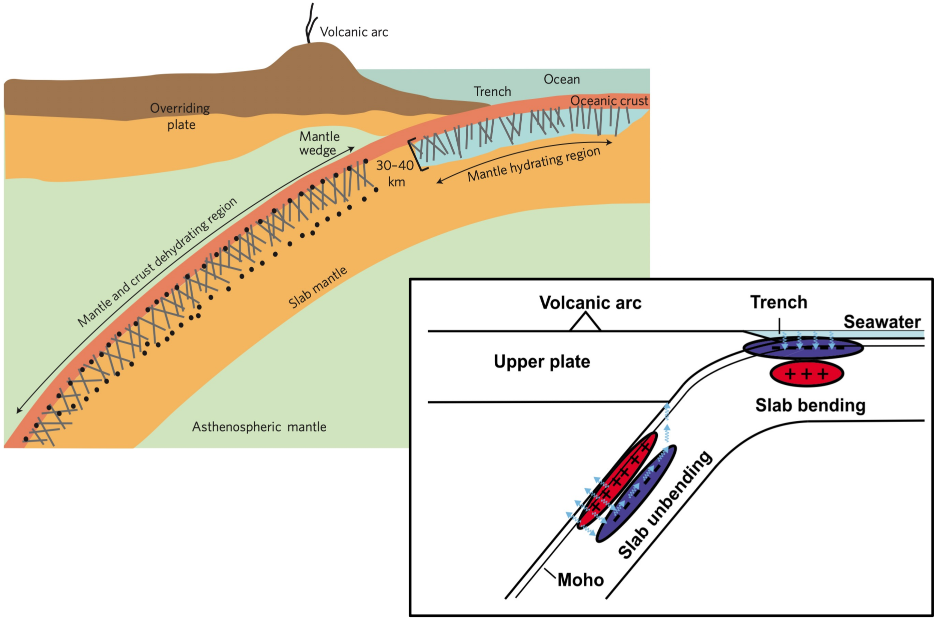


Subducting slabs bend and develop faults as they sink into the mantle, acting as

conduits for water

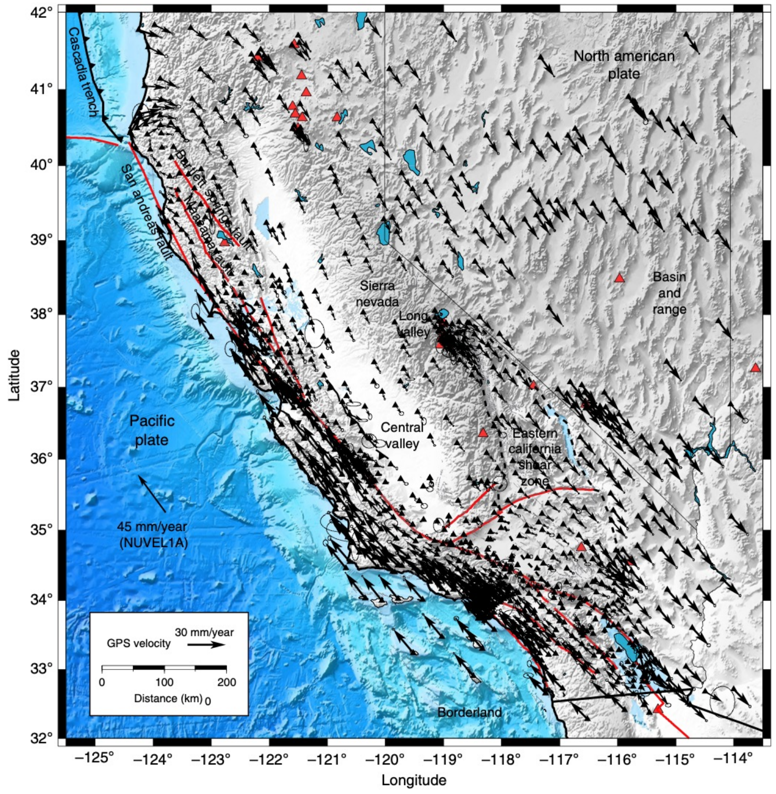
Geodynamic examples: bending of oceanic plates





Numerical modelling indicates that pressure gradients resulting from the bending of slabs may then drive the water deep into their interior.

GPS strain rate maps: California



How GPS strain rates are computed

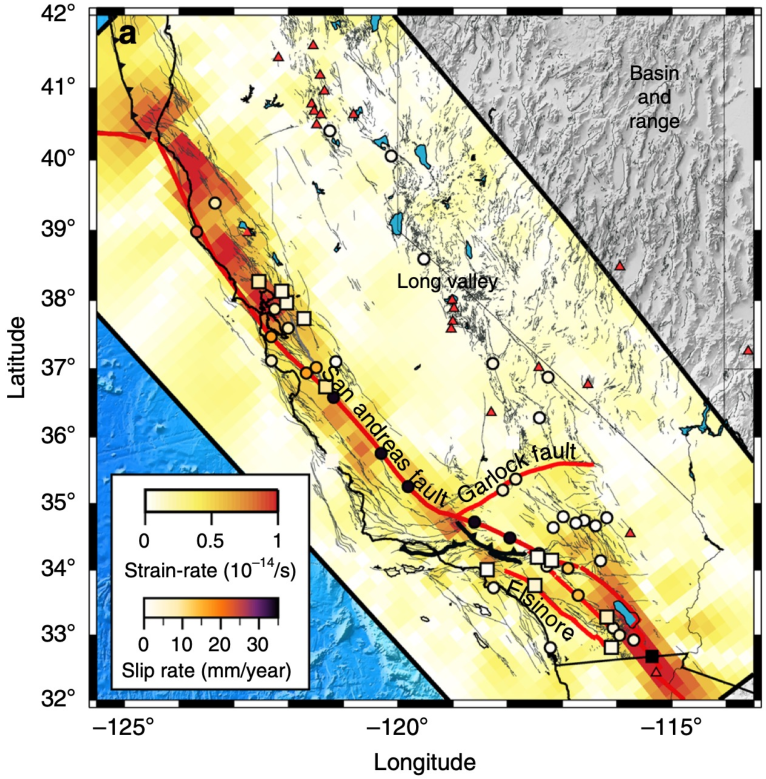
GPS stations measure horizontal velocities with mm/yr precision. Velocities interpolated across a region, using methods such as:

- least-squares interpolation
- triangular tessellation
- spline fitting
- or continuous deformation models

Compute strain-rate tensor:

$$\dot{\varepsilon}_{ij} = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right)$$

GPS strain rate maps: California



What GPS strain rate maps tell us

- Present-day deformation
- Where the crust is extending (red)
- Where the crust is compressing (blue)
- Locations of shear zones

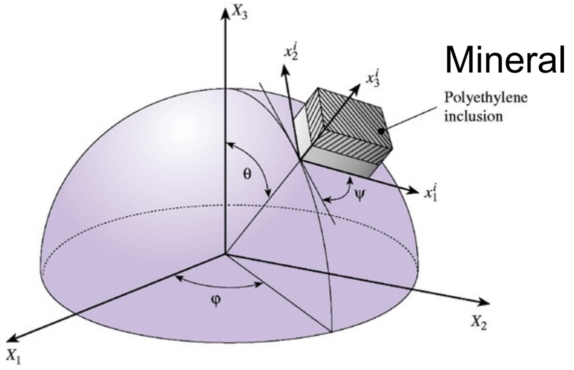
Lithospheric rheology, high strain rates imply:

- Weak crust or warm lithosphere
- Presence of faults or shear zones
- High geodynamic forcing

Seismic anisotropy as a proxy for mantle flow fabric

The elastic properties of solid rocks depend on the modal abundance and orientation of the mineral phases

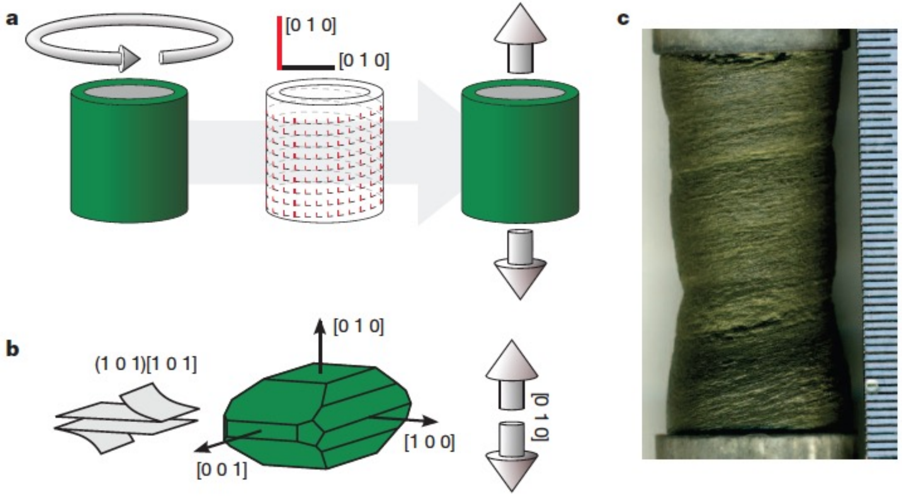
Symmetry	# elastic constants
Triclinic	21
Monoclinic	13
Orthorombic	9
Tetragonal	6
Hexagonal	5
Cubic	3
Isotropic	2



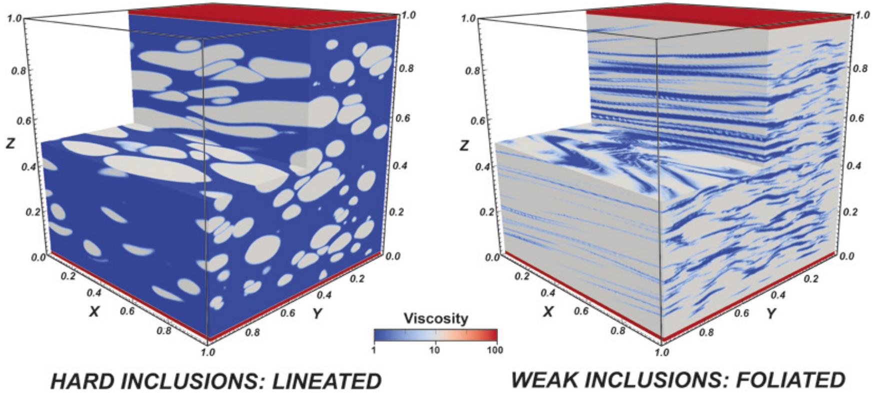
A mechanically isotropic rock is equivalent to a rock with crystal random distribution

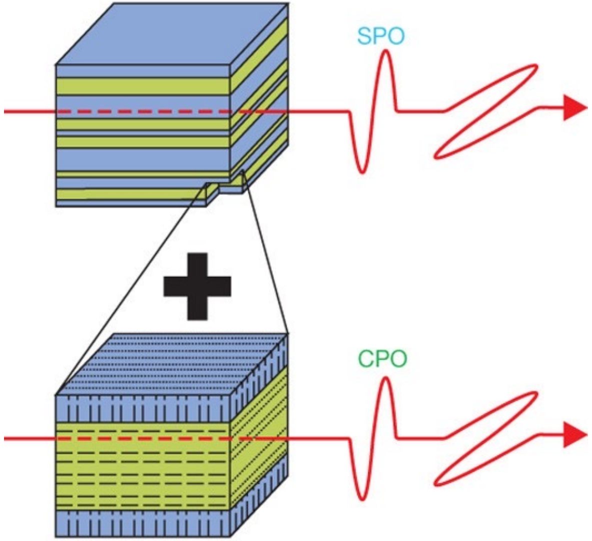
Mechanical anisotropy induced by dislocation creep

Single crystals are mechanically anisotropic during movement of dislocations. Because intra-crystalline deformation occurs along specific planes, dislocation creep produces crystal rotation and preferential alignment of crystal, with the easy-slip plane oriented in the direction of flow. Dislocation creep, thus, introduces a fabric or Crystallographic Preferred Orientation (CPO) such that the rock become mechanically anisotropic.



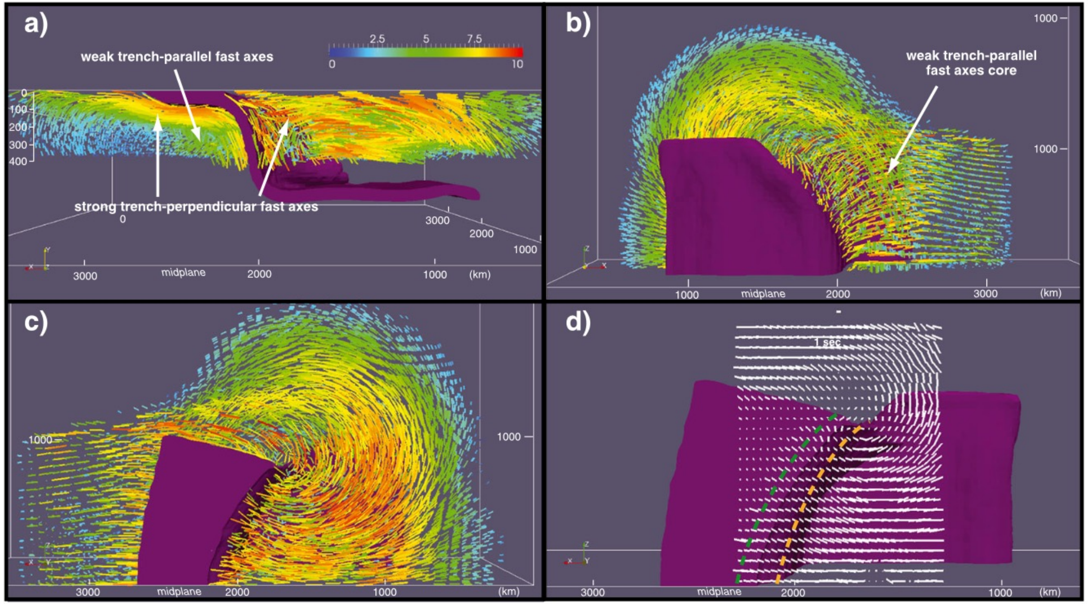
Seismic Anisotropy & Mantle Flow Fabric





Seismic anisotropy means seismic waves travel at different speeds depending on direction. This happens when mantle minerals, mainly olivine, are aligned by deformation. Because olivine crystals have a strong intrinsic anisotropy, their alignment (“lattice-preferred orientation,” LPO) becomes a tracer of mantle flow. ***More to come in the next lectures…***

Seismic Anisotropy & Mantle Flow Fabric



Seismic anisotropy around subduction zones often shows a distinct pattern of poloidal and toroidal mantle flow: in the mantle wedge above the slab, fast axes typically align with poloidal corner flow, where material is drawn downward with the slab and then circulates upward. Around the slab edges, anisotropy instead reflects toroidal flow, where the mantle rolls around the slab tip in a horizontal, doughnut-like pattern, evidence that slabs strongly redirect asthenospheric flow in three dimensions.